# Package 'BayesMassBal'

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Type Package

Title Bayesian Data Reconciliation of Separation Processes

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**Description** Bayesian tools that can be used to reconcile, or mass balance, mass flow rate data collected from chemical or particulate separation processes aided by constraints governed by the conservation of mass.

Functions included in the package aid the user in organizing and constraining data, using Markov chain Monte Carlo methods to obtain samples from Bayesian models, and in computation of the marginal likelihood of the data, given a particular model, for model selection. Marginal likelihood is approximated by methods in Chib S (1995) <doi:10.2307/2291521>.

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**BMB** 

Bayesian Mass Balance

### **Description**

Allows the user to specify the covariance structure for a Bayesian mass balance, simulates draws from reconciled masses and relevant covariance matrix, and approximates the log-marginal likelihood.

# Usage

```
BMB(
    X,
    y,
    cov.structure = c("indep", "component", "location"),
    priors = "default",
    BTE = c(500, 20000, 1),
    lml = FALSE,
    ybal = TRUE,
    diagnostics = TRUE,
    verb = 1
)
```

# **Arguments**

У

X A matrix that maps constrained masses to observed masses. Can be built from the function constrainProcess, see documentation for details.

A list of matrices of observed mass flow rates. Each matrix is a separate sample component. The rows of each matrix index the sampling location, and the columns index the sample set number. Can be specified using the importObservations function.

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cov.structure Character string. "indep" allows for no correlation. "component" indicates

correlation within an individual sample component. "location" indicates correlation within an individual sampling location. Not specifying  ${\tt cov.structure}$ 

defaults to the "indep" structure.

priors List or character string. When the default value priors = "default" is used, the

BMB uses a set of default conjugate priors. When passing a list to the argument, the list must contain user specified hyperparameter values for each conjugate prior. To see the required list structure run BMB with BTE = c(1,2,1) and inspect the output. When priors = "Jeffreys" the Jeffreys priors for  $\Sigma$  and  $\sigma^2$  with known mean given in (Yang and Berger 1996). When priors = "Jeffreys", the prior used for  $\beta$  is proportional to the indicator function  $I[\beta > 0]$ . See

Details for more information.

BTE Numeric vector giving c(Burn-in, Total-iterations, and Every) for MCMC

approximation of target distributions. The function BMB produces a total number of samples of (T-B)/E. E specifies that only one of every E draws are saved. E>1 reduces autocorrelation between obtained samples at the expense

of computation time.

lml Logical indicating if the log-marginal likelihood should be approximated. De-

fault is FALSE, which reduces computation time. Log-marginal likelihood is

approximated using methods in (Chib 1995).

ybal Logical indicating if the mass balanced samples for each y should be returned.

Default is TRUE. Setting ybal=FALSE results in a savings in RAM and computa-

tion time.

diagnostics Logical or list indicating if diagnostic functions geweke.diag and effectiveSize

(Plummer et al. 2006) should computed for the obtained samples. The default of TRUE indicates diagnostics should be run with their default parameters. Alternatively, passing a list of the structure list(frac1 = 0.1, frac2 = 0.5) will run both diagnostics and allow geweke.diag to be run with parameters other

than the default.

verb Numeric indicating verbosity of progress printed to R-console. The default of 1

prints messages and a progress bar to the console during all iterative methods.

verb = 0 indicates no messages are printed.

#### **Details**

See vignette("Two\_Node\_Process", package = "BayesMassBal") for further details on how to use function outputs.

When the priors argument is left unspecified, a set of default conjugate priors are used, which are chosen to allow BMB() to work well in a general setting. In the current version of the BayesMassBal package, only the conjugate priors stated below can be used, but hyperparameter values can be specified by the user using the priors argument.

The prior distribution on beta is a normal distribution truncated at 0. The mean of this distribution before truncation is the ordinary least squares (OLS) estimate of  $\beta$ . OLS estimates less than 0, are changed to 0. The prior variance, before truncation, of each element of  $\beta$  is set to:

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Currently, there is only support for diagonal prior covariance matrices for  $\beta$ 

When cov.structure = "indep" the error of all observations in a sample set are independent. An inverse gamma prior distribution, with  $\alpha_0 = 0.000001$  and  $\beta_0 = 0.000001$ , is placed on the variance of the mass flow rate for each sample component at each sample location.

When cov. structure = "component" or "location", the prior distribution on  $\Sigma_i$  is inverse Wishart  $(\nu_0, \nu_0 \times S_0)$ . The degrees of freedom parameter,  $\nu_0$ , is equal to the dimension of  $\Sigma_i$ . The scale matrix parameter is equal to a matrix,  $S_0$ , with the sample variance of the relevant observation on the diagonal, multiplied by  $\nu_0$ .

The user is able to specify the prior hyperparameters of the mean and variance of beta,  $\alpha_0$  and  $\beta_0$  for each  $\sigma^2$ , and the degrees of freedom and scale matrix for each  $\Sigma_i$  using the priors argument. It is advisable for the user to specify their own prior hyperparameters for  $p(\sigma^2)$  if the variance of any element is well under 1, or  $p(\beta)$  if the there is a wide range in the magnitude of observations.

When priors = "Jeffreys" Jeffreys priors are used for the prior distribution of the variance and covariance parameters. Priors used are  $p(\sigma^2) \propto \frac{1}{\sigma^2}$  and  $p(\Sigma) \propto |\Sigma|^{-(p+1)/2}$ , as listed in (Yang and Berger 1996). The Jeffreys prior for a  $\beta$  with infinite support is  $p(\beta) \propto 1$ . To preserve the prior information that  $\beta>0$ ,  $p(\beta) \propto I[\beta>0]$  is chosen. It is not possible to calculate log-marginal likelihood using the methods in (Chib 1995) with Jeffreys priors. Therefore, if priors = "Jeffreys" and lml = TRUE, the lml argument will be ignored and a warning will be printed.

lml is reported in base e. See here for some guidance on how to interpret Bayes Factors, but note log base log is used on Wikipedia.

#### Value

Returns a list of outputs

diagnostics

beta	List of matrices of samples from the distribution of reconciled data. Each matrix in the list is a separate sample component. Each column of a matrix in beta is a draw from the target distribution.
Sig	List of matrices containing draws from the distribution of each covariance matrix. If S.t is the $t^{th}$ draw from the distribution of covariance matrix S and:
	<ul> <li>cov.structure = "indep", the t<sup>th</sup> column of a matrix in Sig is diag(S.t).</li> <li>cov.structure = "component" or "location", the t<sup>th</sup> column of a matrix in Sig is equal to S.t[upper.tri(S.t, diag = TRUE)].</li> </ul>
priors	List of prior hyperparameters used in generating conditional posterior distributions and approximating log-marginal likelihood. The structure of the input argument priors is required to be the same as the structure of this returned list slice. See Details.
cov.structure	Character string containing the covariance structure used.
y.cov	List of character matrices indicating details for the structure of each covariance matrix. Only returned when cov.structure = "location"
lml	Numeric of the log-marginal likelihood approximation. Returns NA when $lml = FALSE$

List containing results from diagnostic functions geweke.diag and effectiveSize

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ybal	List of samples from the distribution of reconciled mass flow rates, in the same format as the function argument y. Produced with argument ybal = TRUE. Equivalent to lapply(BMB()\$beta, function( $X,x$ ){ $x ** X$ }, $x = X$ ). Viewing this output is more intuitive than viewing samples of beta, at the expense of RAM and some computation time.
X	The function argument X is passed to the output so that it can be used with other BayesMassBal functions.
type	Character string used by plot.BayesMassBal. type = "BMB" for an object returned from the BMB function.

#### References

Chib S (1995). "Marginal likelihood from the Gibbs output." *Journal of the american statistical association*, **90**(432), 1313–1321. Casella G, George EI (1992). "Explaining the Gibbs sampler." *The American Statistician*, **46**(3), 167–174. Plummer M, Best N, Cowles K, Vines K (2006). "CODA: Convergence Diagnosis and Output Analysis for MCMC." *R News*, **6**(1), 7–11. https://journal.r-project.org/archive/. Yang R, Berger JO (1996). *A catalog of noninformative priors*. Institute of Statistics and Decision Sciences, Duke University.

### **Examples**

constrainProcess

Matrix Constraining Process

# Description

Generates matrix X which maps constrained masses  $\beta$  to observed masses y for an individual sample component, when given a linear system of constraining equations.

## Usage

```
constrainProcess(C = NULL, file = FALSE)
```

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#### **Arguments**

C Matrix of constraints for a process at steady state. See Details below.

Character string indicating file path for a \*. csv file containing linear constraints.

Only values of -1, 0, and 1 are valid. The first row in the file is required to be

a header naming the sampling locations. See Details for an example.

#### **Details**

The output of this function is meant to be used as the input parameter X in the BMB function. The matrix C, or imported matrix from file, indexes sampling locations via columns, and number of constraints via rows. Only values of -1, 0, and 1 are valid, and indicate mass leaving a node, a location that is not relevant to a node, and mass entering a node respectively. Constraints should only be indicated around each node. No additional, and no less constraints should be specified. Additional constraints are redundant. Each sample component is subject to the same constraints, and therefore the constraints given to constrainProcess do not need to be repeated for each component.

See vignette("Two\_Node\_Process") for an application example.

The file path of a \*.csv file, which could be used to indicate the constraints for the process in vignette("Two\_Node\_Process"), can be found by typing system.file("extdata", "twonode\_constraints.csv",pacl = "BayesMassBal"), and can be used as a template for other processes.

#### Value

Returns the matrix X which maps  $\beta$  to observed masses y. No changes need to be made to X when using with BMB.

```
## For a single node process where
## y_1 = y_2 + y_3

C <- matrix(c(1,-1,-1), nrow= 1,ncol = 3)
constrainProcess(C = C)

## For a 2 node process with 1 input and 3 outputs
## as shown in \dontrun{vignette("Two_Node_Process", package = "BayesMassBal")}

C <- matrix(c(1,-1,0,-1,0,0,1,-1,0,-1), byrow = TRUE, ncol = 5, nrow = 2)
constrainProcess(C = C)

## Obtaining the constraints from a .csv file

C <- constrainProcess(file =
    system.file("extdata", "twonode_constraints.csv",package = "BayesMassBal"))</pre>
```

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importObservations	Import Observed Mass Flow Rates

#### **Description**

Imports observed mass flow rates stored in a \*.csv file and then organizes the data for use with the BMB function.

# Usage

```
importObservations(file, header = TRUE, csv.params = NULL)
```

#### **Arguments**

file Character string containing the name of \*.csv from file which data will be read.

See Details below for valid file structures.

header Logical indicating if the first row of file file contains header information. Cur-

rent implementation of importObservations discards this information.

csv.params List of arguments to be passed to read.csv

#### **Details**

The purpose of this function is to make it easy to import and structure loosely organized data contained in a \*.csv into a list for use as the y argument passed to the BMB function. The entries in file must be organized as such:

- The first column of file must contain an integer sample location. The value of this integer must correspond to the column number used to specify linear constraints in constrainProcess. For example, data for a given component collected at sampling location  $y_2$  should be indicated with a 2 in the first column of file used with importObservations. In the file used with constrainProcess, the linear constraint(s) on  $y_2$  are indicated in the second column.
- The second column of file must contain sample component names. **This field is case sensitive**. Ensure a given sample component is named consistently, including capitalization and spacing.
- Columns 3 to K+2 of file must contain observed mass flow rates for the K collected sample sets. All observations located in the same column should be collected at the same time.
- Sample components of interest must be specified for each location. If a sample component is not detected at some locations, but is detected at others, this component should be included in file with a specified mass flow rate of 0, or a very small number.

importObservations reads the contents of file, sorts the sampling locations numerically, then creates a list of data frames. Each data frame contains the data for a single sample component.

#### Value

Returns a list of data frames. Each data frame is named according to the unique sample components specified in the second column of file. This list object is intended to be used as the argument y for the BMB function.

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# **Examples**

mainEff

Main Effects

# **Description**

Calculates the main effect of a variable, which is independent of process performance, on a function.

# Usage

```
mainEff(
   BMBobj,
   fn,
   rangex,
   xj,
   N = 50,
   res = 100,
   hdi.params = c(1, 0.95),
   ...
)
```

# **Arguments**

BMBobj	A BayesMassBal object originally obtained from the BMB function. See BMB.
fn	A character string naming a function with arguments of BMBobj\$ybal and independent random variables X. See Details and examples for more on function requirements.
rangex	A numeric matrix. Each column of rangex contains the minimum and maximum value of uniformly distributed random values making up vector $\boldsymbol{x}$ .
xj	Integer indexing which element in $x$ is used for conditioning for $E_x[f(x,y) x_j]$ . If a vector is supplied the marginal main effect of each element is calculated sequentially. The integers supplied in xj are equivalent to the indices of the columns in rangex.
N	Integer specifying the length of the sequence used for xj. Larger N trades a higher resolution of the main effect of xj for longer computation time and larger RAM requirements.

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res Integer indicating the number of points to be used for each Monte-Carlo integration step. Larger res reduces Monte-Carlo variance as the expense of computation time.

hdi.params Numeric vector of length two, used to calculate Highest Posterior Density In-

terval (HPDI) of the main effect xj using hdi. hdi.params[1] = 1 indicates hdi is used, and the mean and HPDI bounds are returned instead of the every sample from the distribution of  $E_x[f(x,y)|x_j]$ . The second element of hdi is passed to the credMass argument in the hdi function. The default, hdi.params

= c(1,0.95), returns 95% HPDI bounds.

... Extra arguments passed to the named fn

#### **Details**

The mainEff function returns a distribution of  $E_x[f(x,y)|x_j]$ , marginalized over the samples of BMBobj\$ybal, giving the distribution of  $E_x[f(x,y)|x_j]$  which incorporates uncertainty of a chemical or particulate process.

In the current implementation of mainEff in the BayesMassBal package, only uniformly distributed values of x are supported.

The f(x,y) is equivalent to the supplied function named in mainEff(fn). For the arguments of fn, ybal is structured in a similar manner as BMBobj\$ybal. The only difference being individual columns of each matrix are used at a time, and are vectorized. Note the way ybal is subset in the example function fn\_example. The supplied X is a matrix, with columns corresponding to each element in x. The output to fn must be a vector of length nrow(x). The first argument of fn must be X, the second argument must be BMBobj\$ybal. Order of other arguments passed to fn through . . . does not matter. Look at the example closely for details!

#### Value

A list of length(xj) list(s). Each list specifies output for the main effect of a xj

g The grid used for a particular xj fn.out A matrix giving results on  $E_x[f(x,y)|x_j]$ . If hdi.params[1] = 1, the mean and Highest Posterior Density Interval (HPDI) bounds of  $E_x[f(x,y)|x_j]$  are returned. Otherwise, samples of  $E_x[f(x,y)|x_j]$  are returned. The index of each column of fn.out corresponds to the the value of g at the same index.

fn Character string giving the name of the function used. Same value as argument

fn.

xj Integer indicating the index of x corresponding to a grouped fn.out and g.

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```
X <- constrainProcess(C = C)</pre>
BMB_example <- BMB(X = X, y = y, cov.structure = "indep",
                    BTE = c(10, 200, 1), lml = FALSE, verb=0)
fn_example <- function(X,ybal){</pre>
    cu.frac <- 63.546/183.5
    feed.mass <- ybal$CuFeS2[1] + ybal$gangue[1]</pre>
    ## Concentrate mass per ton feed
    con.mass <- (ybal$CuFeS2[3] + ybal$gangue[3])/feed.mass</pre>
    ## Copper mass per ton feed
    cu.mass <- (ybal$CuFeS2[3]*cu.frac)/feed.mass</pre>
    gam <- c(-1,-1/feed.mass,cu.mass,-con.mass,-cu.mass,-con.mass)</pre>
    f <- X %*% gam
    return(f)
rangex <- matrix(c(4.00 ,6.25,1125,1875,3880,9080,20,60,96,208,20.0,62.5),
                   ncol = 6, nrow = 2)
mE_example <- mainEff(BMB_example, fn = "fn_example", rangex = rangex, xj = 3, N = 15, res = 4)
```

plot.BayesMassBal

Plots BayesMassBal Object

#### **Description**

Visualizes data from a BayesMassBal class object in a user specified way. Options include trace plots, posterior densities, and main effects plots. Meant to be a quick diagnostic tool, and not to produce publication quality plots.

# Usage

```
## $3 method for class 'BayesMassBal'
plot(
    x,
    sample.params = NA,
    layout = c("trace", "dens"),
    hdi.params = c(1, 0.95),
    ssEst.ylab = "Mass",
    ...
)
```

#### **Arguments**

x A BayesMassBal object returned from the BMB function

sample.params

List to be used for indicating model parameter samples used for creation of plot(s). See details for required structure.

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Character string indicating the desired data to be plotted. "trace" produces trace plots of sequential parameter draws. "dens" produces densities of posterior draws. Argument ignored when x\$type = "time-series".

hdi.params

Numeric vector of length two, used to draw Highest Posterior Density Intervals (HPDI) using hdi, and otherwise ignored. hdi.params[1] = 1 indicates hdi bounds should be drawn. The second element of hdi is passed to credMass in the hdi function. The default, hdi.params = c(1,0.95), plots the 95% HPDI bounds.

ssEst.ylab

Character string providing the label for the y-axis of a time series plot when object type == "time-series". Argument only useful with the output from the ssEst function.

Passes extra arguments to plot()

#### **Details**

The list of sample.params requires a specific structure dependent on the choice of layout and the desired plots.

If layout = "trace" or layout = "dens", names(list) must contain each model parameter desired for plotting. The structure under the model parameter names must be the same as to the structure of the relevant subset of the BayesMassBal object to be used. For example, if a BayesMassBal object is created using a process with sample components c("CuFeS2", "gangue") and the users wants plots of reconciled masses  $y_1$  and  $y_2$  for both components to be created, params = list(y.bal = list(CuFeS2 = c(1,2), gangue = c(1,2)) should be used. Note, str(params) mimics str(x), while the vectors listed simply index the desired model parameters to be plotted.

See vignette("Two\_Node\_Process", package = "BayesMassBal") for an example of the required structure.

### Value

Plots BayesMassBal object based on arguments passed to plot.

pointmassbal	Point Estimate Mass Balance

### **Description**

Function conducting a two component, two node, point estimate mass balance from (Wills 2006) on a two node process. This function is provided for the purpose of comparing the performance of a Bayesian mass balance to a point estimate mass balance using any output of two nodeSim.

#### Usage

pointmassbal(y)

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# **Arguments**

У

A list of matrices of observed mass flow rates. Each matrix is a separate sample component. The rows of each matrix index the sampling location, and the columns index the sample set number. Necessary to format exactly the same as the output of twonodeSim()\$simulation. Any arguments to twonodeSim can be used.

#### Value

Returns a list of vectors with mass flow rates yhat and grades ahat. Similar format to argument y. The index of a vector in the output is equivalent to the index of a row in y.

#### References

Wills BA (2006). *Mineral processing technology: an introduction to the practical aspects of ore treatment and mineral recovery*. Butterworth-Heinemann, Oxford Boston. ISBN 978-0-7506-4450-1.

# **Examples**

```
y <- twonodeSim()$simulation
yhat <- pointmassbal(y)$yhat</pre>
```

ssEst

Steady State Estimate

#### **Description**

Allows for the estimation of process steady state of a single stream for a process using flow rate data.

#### Usage

```
ssEst(y, BTE = c(100, 1000, 1), stationary = FALSE)
```

#### **Arguments**

у	Vector of mass flow rate observations. Must be specified sequentially with y[1] as the initial observation.
BTE	Numeric vector giving c(Burn-in, Total-iterations, and Every) for MCMC approximation of target distributions. The function BMB produces a total number of samples of $(T - B)/E$ . $E$ specifies that only one of every $E$ draws are
	saved. $E > 1$ reduces autocorrelation between obtained samples at the expense

of computation time.

stationary Logical indicating if stationarity will be imposed when generating posterior

draws. See Details.

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#### **Details**

The model of the following form is fit to the data:

$$y_t = \mu + \alpha y_{t-1} + \epsilon$$

Where  $\epsilon \sim \mathcal{N}(0, \sigma^2)$  and t indexes the time step.

A time series is stationary, and predictable, when  $|\alpha| < 1$ . Stationarity can be enforced, using the argument setting stationary = TRUE. This setting utilizes the priors  $p(\alpha) \sim \mathcal{N}(0, 1000)$  truncated at (-1,1), and  $p(\mu) \sim \mathcal{N}(0, \text{var}(y)*100)$  for inference, producing a posterior distribution for  $\alpha$  constrained to be within (-1,1).

When fitting a model where stationarity is not enforced, the Jeffreys prior of  $p(\mu, \alpha) \propto 1$  is used.

The Jeffreys prior of  $p(\sigma^2) \propto 1/\sigma^2$  is used for all inference of  $\sigma^2$ 

A stationary time series will have an expected value of:

$$\frac{\mu}{1-\alpha}$$

Samples of this expectation are included in the output if stationary = TRUE or if none of the samples of  $\alpha$  lie outside of (-1,1).

The output list is a BMB object, passing the output to plot.BayesMassBal allows for observation of the results.

#### Value

Returns a list of outputs

samples List of vectors containing posterior draws of model parameters

stationary Logical indicating the setting of the stationary argument provided to the ssEst function

y Vector of observations initially passed to the ssEst function.

type Character string giving details of the model fit. Primarily included for use with plot.BayesMassBal

```
## Generating Data
y <- rep(NA, times = 21)

y[1] <- 0
mu <- 3
alpha <- 0.3
sig <- 2
for(i in 2:21){
  y[i] <- mu + alpha*y[i-1] + rnorm(1)*sig
}</pre>
```

```
## Generating draws of model parameters
fit <- ssEst(y, BTE = c(100,500,1))</pre>
```

summary.BayesMassBal Summary of BayesMassBal Object

# Description

Prints a summary table containing mean values and 95

# Usage

```
## S3 method for class 'BayesMassBal'
summary(object, export = NA, ...)
```

# Arguments

object	A BayesMassBal object returned from the BMB function
export	Optional character string specifying location to save a *.csv file containing summary data. Only data related to mass flow rates is printed.
•••	Additional arguments affecting the summary produced. Not used for a BayesMassBal object.

# **Details**

Current implementation only returns statistics for balanced mass flow rates, taken from x\$yba1, and not statistics on  $\beta$  or variance parameters of  $\sigma^2$  and  $\Sigma$ .

The header entry of the table 95% LB should be interpreted as the lower bound of the 95

### Value

A summary table printed to the console, and optionally a saved  $\star$ . csv file saved within the path as specified.

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twonodeSim

Two Node Process Data Simulation

#### **Description**

Simulates data for a stochastic two node, two component process at steady state. Location indices are the same as what is shown in vignette("Two\_Node\_Process", package = "BayesMassBal").

#### Usage

```
twonodeSim(
  K = 7,
  feed = list(rate = 100, sd = 6, CuFeS2grade = 1.2),
  rec = list(CuFeS2 = list(mean = c(98, 95)/100, var = c(5e-05, 8e-05)), gangue =
    list(mean = c(7, 4)/100, var = c(5e-05, 2.5e-05))),
  assayNoise = list(CuFeS2 = c(0.15, 0.2, 0.05, 5e-05, 0.005), gangue = c(5, 1, 0.03,
        2, 0.5)),
  truncation = TRUE
)
```

#### **Arguments**

K Numeric specifying the number of sample sets to be simulated.

feed List specifying qualities for the process grade. See default for required struc-

ture. rate is the mean feed rate. sd is the standard deviation of the feed rate. CuFeS2grade is the mass percent CuFeS2 present in the feed. Grade is not

stochastic. See Details for important information on specifying these values.

rec List specifying mean and variance in process performance. See default for re-

quired structure. rec\$component\$mean is a vector giving mean fractional recovery of the given component for c(node1, node2). rec\$component\$var gives

the variance in the process in a similar manner. See Details.

assayNoise List specifying standard deviations of random noise added to simulated process

outputs. See default for required structure. The index of a vector within the list is equivalent to the index of the sampling location. See Details section for

important information on specifying these values.

truncation Logical indicating if the simulation should be rerun, and previous results dis-

carded, until no observed values are less than 0. Default is TRUE. See details

for more information.

#### **Details**

Each of the K data sets collected from the twonodeSim() simulation is independent and identically distributed.

The feed rate to the process is normally distributed with a mean of feed\$rate, and a standard deviation of feed\$sd. If the feed rate is sufficiently small, and the standard deviation is sufficiently large, negative feed rates can be generated.

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Process recovery at each node is simulated from a beta distribution, reparameterized as shown in this post to make parameter specification more intuitive. This reparameterization is only valid when  $\sigma^2 \le \mu(1-\mu)$ , and the list argument rec must be specified as such.

The steps of the simulation for each sample set are:

- 1. Draw a random normally distributed feed rate.
- 2. Draw random values for recovery of the two components at each node.
- 3. Calculate mass flow rate at each location. These mass flow rates are the *true* mass flow rates, given the process variability.
- 4. Adds normally distributed noise to each observation as specified in argument assayNoise

# If the standard deviations supplied to feed and assayNoise are sufficiently large, the simulation can return negative mass flow rates.

The argument truncation = TRUE discards negative mass flow rates, and reruns the simulation until all values are non-negative. For some combinations of a large K and specifications in feed and assayNoise, this can happen frequently. If if the simulation is run three or more times a warning will be printed that the returned expectations are unreliable. If this is the case, expectations should be calculated using analytical or Monte-Carlo methods outside of the abilities of this function. For the default parameters, truncation can occur, but is rare. The default parameters were chosen in a way that makes a truncation warning highly unlikely.

#### Value

Returns a list of simulated data and expected values. List members are as follows:

simulation List of matrices giving simulated data. twonodeSim()\$simulation is structured so that it can directly be passed to the BMB function as the y argument.

expectations List of matrices giving expected values of the mass flow rate for each component

at every location. See the Details section for information about instances that

may create reliability issues with this output.

```
y <- twonodeSim()$simulation  
## Then the BMB function can be run as C <- matrix(c(1,-1,0,-1,0,0,1,-1,0,-1), byrow = TRUE, ncol = 5, nrow = 2) X <- constrainProcess(C = C)  
BMB(X = X, y = y, BTE = c(100,600,1))
```

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