Package ‘mixbox’

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Type Package

Title Observed Fisher Information Matrix for Finite Mixture Model

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Description Developed for the following tasks. 1- simulating realizations from the canonical, restricted, and unrestricted finite mixture models. 2- Monte Carlo approximation for density function of the finite mixture models. 3- Monte Carlo approximation for the observed Fisher information matrix, asymptotic standard error, and the corresponding confidence intervals for parameters of the mixture models sing the method proposed by Basford et al. (1997) <https://espace.library.uq.edu.au/view/UQ:57525>.

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`AIS` *AIS data*

Description

The set of AIS data involves recorded body factors of 202 athletes including 100 women 102 men, see Cook (2009). Among factors, two variables body mass index (BMI) and body fat percentage (Bfat) are chosen for cluster analysis.

Usage

```
data(AIS)
```

Format

A text file with 3 columns.

References

R. D. Cook and S. Weisberg, (2009). *An Introduction to Regression Graphics*, John Wiley & Sons, New York.

Examples

```
data(AIS)
```

`bankruptcy` *bankruptcy data*

Description

The bankruptcy dataset involves ratio of the retained earnings (RE) to the total assets, and the ratio of earnings before interests and the taxes (EBIT) to the total assets of 66 American firms, see Altman (1969).

Usage

```
data(bankruptcy)
```

Format

A text file with 3 columns.

References

E. I. Altman, 1969. Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *The Journal of Finance*, 23(4), 589-609.

Examples

```
data(bankruptcy)
```

---

dmix *Approximating the density function of the finite mixture models applied for model-based clustering.*

Description

The density function of a G-component finite mixture model can be represented as

$$g(\boldsymbol{y}|\Psi) = \sum_{g=1}^{G} \omega_g f_{\boldsymbol{Y}}(\boldsymbol{y}, \Theta_g),$$

where $\boldsymbol{\Psi} = \big(\boldsymbol{\Theta}_1, \cdots, \boldsymbol{\Theta}_G\big)^\top$ with $\boldsymbol{\Theta}_g = \big(\boldsymbol{\omega}_g, \boldsymbol{\mu}_g, \Sigma_g, \boldsymbol{\lambda}_g\big)^\top$. Herein, $f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g)$ accounts for the density function of random vector $\boldsymbol{Y}$ within each component. In the restricted case, $f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g)$ admits the representation given by

$$\boldsymbol{Y} \stackrel{d}{=} \boldsymbol{\mu}_g + \sqrt{W}\boldsymbol{\lambda}_g|Z_0| + \sqrt{W}\Sigma_g^{\frac{1}{2}}\boldsymbol{Z}_1,$$

where $\boldsymbol{\mu}_g \in R^d$ is location vector, $\boldsymbol{\lambda}_g \in R^d$ is skewness vector, Σ_g is a positive definite symmetric dispersion matrix for $g = 1, \cdots, G$. Further, W is a positive random variable with mixing density function $f_W(w|\boldsymbol{\theta}_g)$, $Z_0 \sim N(0, 1)$, and $\boldsymbol{Z}_1 \sim N_d\big(\boldsymbol{0}, \Sigma_g\big)$. We note that W, Z_0, and $\boldsymbol{Z}_1$ are mutually independent. In the canonical or unrestricted case, $f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g)$ admits the representation as

$$\boldsymbol{Y} \stackrel{d}{=} \boldsymbol{\mu}_g + \sqrt{W}\boldsymbol{\Lambda}_g|\boldsymbol{Z}_0| + \sqrt{W}\Sigma_g^{\frac{1}{2}}\boldsymbol{Z}_1,$$

where $\boldsymbol{\Lambda}_g$ is the skewness matrix and random vector $\boldsymbol{Z}_0$ follows a zero-mean normal random vector truncated to the positive hyperplane R^d whose independent marginals have variance unity. We note that in the unrestricted case $\boldsymbol{\Lambda}_g$ is a $d \times d$ diagonal matrix whereas in the canonical case, it is a $d \times q$ matrix and so, random vector $\boldsymbol{Z}_0$ follows a zero-mean normal random vector truncated to the positive hyperplane R^q.

Usage

```
dmix(Y, G, weight, model = "restricted", mu, sigma, lambda, family = "constant",
skewness = "FALSE", param = NULL, theta = NULL, tick = NULL, N = 3000, log = FALSE)
```

Arguments

`Y`	an $n \times d$ matrix of observations.
`G`	number of components.
`weight`	a vector of weight parameters (or mixing proportions).
`model`	it must be `"canonical"`, `"restricted"`, or `"unrestricted"`. By default `model = "restricted"`.

mu	a list of location vectors of G components.
sigma	a list of dispersion matrices of G components.
lambda	a list of skewness vectors of G components. If model is either "canonical" or "unrestricted", then skewness vector must be given in matrix form of appropriate size.
family	name of mixing distribution. By default family = "constant" that corresponds to the finite mixture of multivariate normal (or skew normal) distribution. Other candidates for family name are: "bs" (for Birnbaum-Saunders), "burriii" (for Burr type iii), "chisq" (for chi-square), "exp" (for exponential), "f" (for Fisher), "gamma" (for gamma), "gig" (for generalized inverse-Gaussian), "igamma" (for inverse-gamma), "igaussian" (for inverse-Gaussian), "lindley" (for Lindley), "loglog" (for log-logistic), "lognorm" (for log-normal), "lomax" (for Lomax), "pstable" (for positive α-stable), "ptstable" (for polynomially tilted α-stable), "rayleigh" (for Rayleigh), and "weibull" (for Weibull).
skewness	a logical statement. By default skewness = "FALSE" which means that a symmetric model is fitted to each component (cluster). If skewness = "FALSE", then a skewed model is fitted to each component.
param	name of the elements of $\boldsymbol{\theta}$ as the parameter vector of mixing distribution with density function $f_W(w\|\boldsymbol{\theta})$. By default it is NULL.
theta	a list of maximum likelihood estimator for $\boldsymbol{\theta}$ (parameter vector of the mixing distribution with density function $f_W(w\|\boldsymbol{\theta})$), across G components. By default it is NULL.
tick	a binary vector whose length depends on type of family. The elements of tick are either 0 or 1. If element of tick is 0, then the corresponding element of $\boldsymbol{\theta}$ is not considered in the formula of $f_W(w\|\boldsymbol{\theta})$ for computing the required posterior expectations. If element of tick is 1, then the corresponding element of $\boldsymbol{\theta}$ is considered in the formula of $f_W(w\|\boldsymbol{\theta})$. For instance, if family = "gamma" and either its shape or rate parameter is one, then tick = c(1). This is while, if family = "gamma" and both of the shape and rate parameters are in the formula of $f_W(w\|\boldsymbol{\theta})$, then tick = c(1, 1). By default tick = NULL.
N	an integer number for approximating the $g(\boldsymbol{y}\|\Psi)$. By default $N = 3000$.
log	if log = TRUE, then it returns the log of the density function. By default it is log = FALSE.

Value

Monte Carlo approximated values of mixture model density function.

Author(s)

Mahdi Teimouri

Examples

```
Y <- c(1, 2)
G <- 2
```

```
 weight <- rep( 0.5, 2 )
    mu1 <- rep(  -5, 2 )
    mu2 <- rep(   5, 2 )
 sigma1 <- matrix( c( 0.4, -0.20, -0.20, 0.5 ), nrow = 2, ncol = 2 )
 sigma2 <- matrix( c( 0.5,  0.20,  0.20, 0.4 ), nrow = 2, ncol = 2 )
lambda1 <- c( 5, -5 )
lambda2 <- c(-5,  5 )
     mu <- list( mu1, mu2 )
  sigma <- list( sigma1 , sigma2 )
 lambda <- list( lambda1, lambda2)
    out <- dmix(Y, G, weight, model = "restricted", mu, sigma, lambda, family =
           "constant", skewness = "TRUE", param = "NULL", theta = "NULL", tick =
           NULL, N = 3000)
```

`ofim1` *Computing observed Fisher information matrix for restricted finite mixture model.*

Description

This function computes the observed Fisher information matrix for a given restricted finite mixture model. For this, we use the method of Basford et al. (1997). The density function of each G-component finite mixture model is given by

$$g(\boldsymbol{y}|\Psi) = \sum_{g=1}^{G} \omega_g f_{\boldsymbol{Y}}(\boldsymbol{y}, \Theta_g),$$

where $\boldsymbol{\Psi} = \big(\boldsymbol{\Theta}_1, \cdots, \boldsymbol{\Theta}_G\big)^\top$ with $\boldsymbol{\Theta}_g = \big(\boldsymbol{\omega}_g, \boldsymbol{\mu}_g, \Sigma_g, \boldsymbol{\lambda}_g\big)^\top$. Herein, $f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g)$ accounts for the density function of random vector $\boldsymbol{Y}$ within g-th component that admits the representation given by

$$\boldsymbol{Y} \stackrel{d}{=} \boldsymbol{\mu}_g + \sqrt{W}\boldsymbol{\lambda}_g|Z_0| + \sqrt{W}\Sigma_g^{\frac{1}{2}}\boldsymbol{Z}_1,$$

where $\boldsymbol{\mu}_g \in R^d$ is location vector, $\boldsymbol{\lambda}_g \in R^d$ is skewness vector, Σ_g is a positive definite symmetric dispersion matrix for $g = 1, \cdots, G$. Further, W is a positive random variable with mixing density function $f_W(w|\boldsymbol{\theta}_g)$, $Z_0 \sim N(0, 1)$, and $\boldsymbol{Z}_1 \sim N_d\big(\boldsymbol{0}, \Sigma_g\big)$. We note that W, Z_0, and $\boldsymbol{Z}_1$ are mutually independent. For approximating the observed Fisher information matrix of the finite mixture models, we use the method of Basford et al. (1997). Based on this method, using observations $\boldsymbol{y} = (\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_n)^\top$, an approximation of the expected information

$$-E\Big\{\frac{\partial^2 \log L(\boldsymbol{\Psi})}{\partial\boldsymbol{\Psi}\partial\boldsymbol{\Psi}^\top}\Big\},$$

is give by the observed information as

$$\sum_{i=1}^{n} \hat{\boldsymbol{h}}_i \hat{\boldsymbol{h}}_i^\top,$$

where

$$\hat{\boldsymbol{h}}_i = \frac{\partial}{\partial\boldsymbol{\Psi}} \log L_i(\hat{\boldsymbol{\Psi}})$$

and $\log L(\hat{\boldsymbol{\Psi}}) = \sum_{i=1}^{n} \log L_i(\hat{\boldsymbol{\Psi}}) = \sum_{i=1}^{n} \log\Big\{\sum_{g=1}^{G} \widehat{\omega}_g f_{\boldsymbol{Y}}\big(\boldsymbol{y}_i|\widehat{\boldsymbol{\Theta}_g}\big)\Big\}$. Herein $\widehat{\omega}_g$ and $\widehat{\boldsymbol{\Theta}_g}$ denote the maximum likelihood estimator of ω_g and $\boldsymbol{\Theta}_g$, for $g = 1, \cdots, G$, respectively.

Usage

```
ofim1(Y, G, weight, mu, sigma, lambda, family = "constant", skewness = "FALSE",
     param = NULL, theta = NULL, tick = NULL, h = 0.001, N = 3000, level = 0.05,
   PDF = NULL )
```

Arguments

`Y`	an $n \times d$ matrix of observations.
`G`	number of components.
`weight`	a vector of weight parameters (or mixing proportions).
`mu`	a list of location vectors of `G` components.
`sigma`	a list of dispersion matrices of `G` components.
`lambda`	a list of skewness vectors of `G` components.
`family`	name of the mixing distribution. By default `family = "constant"` that corresponds to the finite mixture of multivariate normal (or skew normal) distribution. Other candidates for family name are: "bs" (for Birnbaum-Saunders), "burriii" (for Burr type iii), "chisq" (for chi-square), "exp" (for exponential), "f" (for Fisher), "gamma" (for gamma), "gig" (for generalized inverse-Gaussian), "igamma" (for inverse-gamma), "igaussian" (for inverse-Gaussian), "lindley" (for Lindley), "loglog" (for log-logistic), "lognorm" (for log-normal), "lomax" (for Lomax), "pstable" (for positive α-stable), "ptstable" (for polynomially tilted α-stable), "rayleigh" (for Rayleigh), and "weibull" (for Weibull).
`skewness`	logical statement. By default `skewness = "FALSE"` which means that a symmetric model is fitted to each component (cluster). If `skewness = "TRUE"`, then a skewed model is fitted to each component.
`param`	name of the elements of θ as the parameter vector of mixing distribution with density function $f_W(w\|\boldsymbol{\theta})$. By default it is `NULL`.
`theta`	a list of maximum likelihood estimator for θ across `G` components. By default it is `NULL`.
`tick`	a binary vector whose length depends on type of family. The elements of `tick` are either `0` or `1`. If element of `tick` is `0`, then the corresponding element of $\boldsymbol{\theta}$ is not considered in the formula of $f_W(w\|\boldsymbol{\theta})$ for computing the required posterior expectations. If element of `tick` is `1`, then the corresponding element of $\boldsymbol{\theta}$ is considered in the formula of $f_W(w\|\boldsymbol{\theta})$. For instance, if `family = "gamma"` and either its shape or rate parameter is one, then `tick = c(1)`. This is while, if `family = "gamma"` and both of the shape and rate parameters are in the formula of $f_W(w\|\boldsymbol{\theta})$, then `tick = c(1, 1)`. By default `tick = NULL`.
`h`	a positive small value for computing numerical derivative of $f_W(w\|\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$, that is $\partial/\partial\theta f_W(w\|\boldsymbol{\theta})$. By default $h = 0.001$.
`N`	an integer number for approximating the posterior expected values within the E-step of the EM algorithm through the Monte Carlo method. By default $N = 3000$.
`level`	significance level α for constructing $100(1-\alpha)\%$ confidence interval. By default $\alpha = 0.05$.
`PDF`	mathematical expression for mixing density function $f_W(w\|\boldsymbol{\theta})$. By default it is `NULL`.

Value

A two-part list whose first part is the observed Fisher information matrix for finite mixture model.

Author(s)

Mahdi Teimouri

References

K. E. Basford, D. R. Greenway, G. J. McLachlan, and D. Peel, (1997). Standard errors of fitted means under normal mixture, *Computational Statistics*, 12, 1-17.

Examples

```
      n <- 100
      G <- 2
 weight <- rep( 0.5, 2 )
    mu1 <- rep(-5  , 2 )
    mu2 <- rep( 5  , 2 )
 sigma1 <- matrix( c(0.4, -0.20, -0.20, 0.5 ), nrow = 2, ncol = 2 )
 sigma2 <- matrix( c(0.5,  0.20,  0.20, 0.4 ), nrow = 2, ncol = 2 )
lambda1 <- c( 5, -5 )
lambda2 <- c(-5,  5 )
     mu <- list( mu1, mu2 )
 lambda <- list( lambda1, lambda2 )
  sigma <- list( sigma1 , sigma2  )
    PDF <- quote( (b/2)^(a/2)*x^(-a/2 - 1)/gamma(a/2)*exp( -b/(x*2) ) )
  param <- c( "a","b")
 theta1 <- c( 10, 12 )
 theta2 <- c( 10, 20 )
  theta <- list( theta1, theta2 )
  tick  <- c( 1, 1 )
      Y <- rmix(n, G, weight, model = "restricted", mu, sigma, lambda, family = "igamma", theta)
    out <- ofim1(Y[, 1:2], G, weight, mu, sigma, lambda, family = "igamma", skewness = "TRUE",
             param, theta, tick, h = 0.001, N = 3000, level = 0.05, PDF)
```

`ofim2` *Computing observed Fisher information matrix for unrestricted or canonical finite mixture model.*

Description

This function computes the observed Fisher information matrix for a given unrestricted or canonical finite mixture model. For this, we use the method of Basford et al. (1997). The density function of each G-component finite mixture model is given by

$$g(\boldsymbol{y}|\Psi) = \sum_{g=1}^{G} \omega_g f_{\boldsymbol{Y}}(\boldsymbol{y}, \Theta_g),$$

where $\boldsymbol{\Psi} = \big(\boldsymbol{\Theta}_1, \cdots, \boldsymbol{\Theta}_G\big)^\top$ with $\boldsymbol{\Theta}_g = \big(\boldsymbol{\omega}_g, \boldsymbol{\mu}_g, \Sigma_g, \boldsymbol{\lambda}_g\big)^\top$. Herein, $f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g)$ accounts for the density function of random vector $\boldsymbol{Y}$ within g-th component that admits the representation given by

$$\boldsymbol{Y} \stackrel{d}{=} \boldsymbol{\mu}_g + \sqrt{W}\boldsymbol{\lambda}_g|Z_0| + \sqrt{W}\Sigma_g^{\frac{1}{2}}\boldsymbol{Z}_1,$$

where $\boldsymbol{\mu}_g \in R^d$ is location vector, $\boldsymbol{\lambda}_g \in R^d$ is skewness vector, Σ_g is a positive definite symmetric dispersion matrix for $g = 1, \cdots, G$. Further, W is a positive random variable with mixing density function $f_W(w|\boldsymbol{\theta}_g)$, $Z_0 \sim N(0,1)$, and $\boldsymbol{Z}_1 \sim N_d\big(\boldsymbol{0}, \Sigma\big)$. We note that W, Z_0, and $\boldsymbol{Z}_1$ are mutually independent. For approximating the observed Fisher information matrix of the finite mixture models, we use the method of Basford et al. (1997). Based on this method, using observations $\boldsymbol{y} = (\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_n)^\top$, an approximation of the expected information

$$-E\Big\{\frac{\partial^2 \log L(\boldsymbol{\Psi})}{\partial \boldsymbol{\Psi} \partial \Psi^\top}\Big\},$$

is give by the observed information as

$$\sum_{i=1}^{n} \hat{\boldsymbol{h}}_i \hat{\boldsymbol{h}}_i^\top,$$

where

$$\hat{\boldsymbol{h}}_i = \frac{\partial}{\partial \boldsymbol{\Psi}} \log L_i(\hat{\boldsymbol{\Psi}})$$

and $\log L(\hat{\boldsymbol{\Psi}}) = \sum_{i=1}^{n} \log L_i(\hat{\boldsymbol{\Psi}}) = \sum_{i=1}^{n} \log\Big\{\sum_{g=1}^{G} \widehat{\omega}_g f_{\boldsymbol{Y}}\big(\boldsymbol{y}_i|\widehat{\boldsymbol{\Theta}_g}\big)\Big\}$. Herein $\widehat{\omega}_g$ and $\widehat{\boldsymbol{\Theta}_g}$ denote the maximum likelihood estimator of ω_g and $\boldsymbol{\Theta}_g$, for $g = 1, \cdots, G$, respectively.

Usage

```
ofim2(Y, G, weight, model, mu, sigma, lambda, family = "constant", skewness = "FALSE",
     param = NULL, theta = NULL, tick = NULL, h = 0.001, N = 3000, level = 0.05,
    PDF = NULL )
```

Arguments

`Y`	an $n \times d$ matrix of observations.
`G`	number of components.
`weight`	a vector of weight parameters (or mixing proportions).
`model`	It must be `"canonical"` or `"unrestricted"`.
`mu`	a list of location vectors of `G` components.
`sigma`	a list of dispersion matrices of `G` components.
`lambda`	a list of skewness vectors of `G` components. If model is either `"canonical"` or `"unrestricted"`, then skewness vactor must be given in matrix form of appropriate size.
`family`	name of the mixing distribution. By default `family = "constant"` that corresponds to the finite mixture of multivariate normal (or skew normal) distribution. Other candidates for family name are: "bs" (for Birnbaum-Saunders), "burriii" (for Burr type iii), "chisq" (for chi-square), "exp" (for exponential), "f" (for Fisher), "gamma" (for gamma), "gig" (for generalized inverse-Gaussian),

"igamma" (for inverse-gamma), "igaussian" (for inverse-Gaussian), "lindley" (for Lindley), "loglog" (for log-logistic), "lognorm" (for log-normal), "lomax" (for Lomax), "pstable" (for positive α-stable), "ptstable" (for polynomially tilted α-stable), "rayleigh" (for Rayleigh), and "weibull" (for Weibull).

`skewness` logical statement. By default `skewness = "FALSE"` which means that a symmetric model is fitted to each component (cluster). If `skewness = "TRUE"`, then a skewed model is fitted to each component.

`param` name of the elements of θ as the parameter vector of mixing distribution with density function $f_W(w|\boldsymbol{\theta})$. By default it is `NULL`.

`theta` a list of maximum likelihood estimator for θ across `G` components. By default it is `NULL`.

`tick` a binary vector whose length depends on type of family. The elements of `tick` are either `0` or `1`. If element of `tick` is `0`, then the corresponding element of $\boldsymbol{\theta}$ is not considered in the formula of $f_W(w|\boldsymbol{\theta})$ for computing the required posterior expectations. If element of `tick` is `1`, then the corresponding element of $\boldsymbol{\theta}$ is considered in the formula of $f_W(w|\boldsymbol{\theta})$. For instance, if `family = "gamma"` and either its shape or rate parameter is one, then `tick = c(1)`. This is while, if `family = "gamma"` and both of the shape and rate parameters are in the formula of $f_W(w|\boldsymbol{\theta})$, then `tick = c(1, 1)`. By default `tick = NULL`.

`h` a positive small value for computing numerical derivative of $f_W(w|\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$, that is $\partial/\partial\theta f_W(w|\boldsymbol{\theta})$. By default $h = 0.001$.

`N` an integer number for approximating the posterior expected values within the E-step of the EM algorithm through the Monte Carlo method. By default $N = 3000$.

`level` significance level α for constructing $100(1-\alpha)\%$ confidence interval. By default $\alpha = 0.05$.

`PDF` mathematical expression for mixing density function $f_W(w|\boldsymbol{\theta})$. By default it is `NULL`.

Value

A two-part list whose first part is the observed Fisher information matrix for finite mixture model.

Author(s)

Mahdi Teimouri

References

K. E. Basford, D. R. Greenway, G. J. McLachlan, and D. Peel, (1997). Standard errors of fitted means under normal mixture, *Computational Statistics*, 12, 1-17.

Examples

```
n <- 100
G <- 2
```

```
 weight <- rep( 0.5, 2 )
    mu1 <- rep(-5  , 2 )
    mu2 <- rep( 5  , 2 )
 sigma1 <- matrix( c(0.4, -0.20, -0.20, 0.5 ), nrow = 2, ncol = 2 )
 sigma2 <- matrix( c(0.5,  0.20,  0.20, 0.4 ), nrow = 2, ncol = 2 )
lambda1 <- diag( c( 5, -5 ) )
lambda2 <- diag( c(-5,  5 ) )
     mu <- list( mu1, mu2 )
 lambda <- list( lambda1, lambda2 )
  sigma <- list( sigma1 , sigma2  )
    PDF <- quote( (b/2)^(a/2)*x^(-a/2 - 1)/gamma(a/2)*exp( -b/(x*2) ) )
  param <- c( "a","b")
 theta1 <- c( 10, 12 )
 theta2 <- c( 10, 20 )
  theta <- list( theta1, theta2 )
  tick  <- c( 1, 1 )
      Y <- rmix(n, G, weight, model = "unrestricted", mu, sigma, lambda, family = "igamma",
            theta)
    out <- ofim2(Y[, 1:2], G, weight, model = "unrestricted", mu, sigma, lambda,
           family = "igamma", skewness = "TRUE", param, theta, tick, h = 0.001, N = 3000,
           level = 0.05, PDF)
```

---

rmix *Generating realization from finite mixture models.*

Description

The density function of a restricted G-component finite mixture model can be represented as

$$\mathcal{M}(\boldsymbol{y}|\boldsymbol{\Psi}) = \sum_{g=1}^{G} \omega_g f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g),$$

where positive constants $\omega_1, \omega_2, \cdots, \omega_G$ are called weight (or mixing proportions) parameters with this properties that $\sum_{g=1}^{G} \omega_g = 1$ and $\boldsymbol{\Psi} = \big(\boldsymbol{\Theta}_1, \cdots, \boldsymbol{\Theta}_G\big)^\top$ with $\boldsymbol{\Theta}_g = \big(\boldsymbol{\omega}_g, \boldsymbol{\mu}_g, \Sigma_g, \boldsymbol{\lambda}_g\big)^\top$. Herein, $f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g)$ accounts for the density function of random vector $\boldsymbol{Y}$ within g-th component that admits the representation given by

$$\mathbf{Y} \stackrel{d}{=} \boldsymbol{\mu}_g + \sqrt{W}\boldsymbol{\lambda}_g|Z_0| + \sqrt{W}\Sigma_g^{\frac{1}{2}}\mathbf{Z}_1,$$

where $\boldsymbol{\mu}_g \in R^d$ is location vector, $\boldsymbol{\lambda}_g \in R^d$ is skewness vector, and Σ_g is a positive definite symmetric dispersion matrix for $g = 1, \cdots, G$. Further, W is a positive random variable with mixing density function $f_W(w|\boldsymbol{\theta}_g)$, $Z_0 \sim N(0,1)$, and $\boldsymbol{Z}_1 \sim N_d\big(\mathbf{0}, \Sigma_g\big)$. We note that W, Z_0, and $\mathbf{Z}_1$ are mutually independent.

Usage

```
rmix(n, G, weight, model = "restricted", mu, sigma, lambda, family = "constant",
        theta = NULL)
```

Arguments

`n`	number of realizations.
`G`	number of components.
`weight`	a vector of weight parameters (or mixing proportions).
`model`	It must be `"canonical"`, `"restricted"`, or `"unrestricted"`. By default `model="restricted"`.
`mu`	a list of location vectors of `G` components.
`sigma`	a list of dispersion matrices of `G` components.
`lambda`	a list of skewness vectors of `G` components. If mixture model is symmetric, then a vector of zeros of appropriate size should be considered for the skewness vector of the corresponding component.
`family`	name of mixing distribution. By default `family = "constant"` that corresponds to the finite mixture of multivariate normal (or skew normal) distribution. Other candidates for family name are: "bs" (for Birnbaum-Saunders), "burriii" (for Burr type iii), "chisq" (for chi-square), "exp" (for exponential), "f" (for Fisher), "gamma" (for gamma), "gigaussian" (for generalized inverse-Gaussian), "igamma" (for inverse-gamma), "igaussian" (for inverse-Gaussian), "lindley" (for Lindley), "loglog" (for log-logistic), "lognorm" (for log-normal), "lomax" (for Lomax), "pstable" (for positive α-stable), "ptstable" (for polynomially tilted α-stable), "rayleigh" (for Rayleigh), and "weibull" (for Weibull).
`theta`	a list of maximum likelihood estimator(s) for θ (parameter vector of mixing distribution) across `G` components. By default it is `NULL`.

Value

a matrix with n rows and $d+1$ columns. The first d columns constitute n realizations from random vector $\boldsymbol{Y} = (Y_1, \cdots, Y_d)^T$ and the last column is the label of realization $\boldsymbol{Y}_i$ (for $i = 1, \cdots n$) indicating the component that $\boldsymbol{Y}_i$ is coming from.

Author(s)

Mahdi Teimouri

Examples

```
 weight <- rep( 0.5, 2 )
    mu1 <- rep(-5  , 2 )
    mu2 <- rep( 5  , 2 )
 sigma1 <- matrix( c( 0.4, -0.20, -0.20, 0.4 ), nrow = 2, ncol = 2 )
 sigma2 <- matrix( c( 0.4,  0.10,  0.10, 0.4 ), nrow = 2, ncol = 2 )
lambda1 <- matrix( c( -4, -2,  2,  5 ), nrow = 2, ncol = 2 )
lambda2 <- matrix( c(  4,  2, -2, -5 ), nrow = 2, ncol = 2 )
 theta1 <- c( 10, 10 )
 theta2 <- c( 20, 20 )
     mu <- list( mu1, mu2 )
  sigma <- list( sigma1 , sigma2 )
 lambda <- list( lambda1, lambda2)
```

```
theta <- list( theta1 , theta2 )
    Y <- rmix( n = 100, G = 2, weight, model = "canonical", mu, sigma, lambda,
         family = "igamma", theta )
```

sefm *Approximating the asymptotic standard error for parameters of the finite mixture models based on the observed Fisher information matrix.*

Description

The density function of each finite mixture model can be represented as

$$\mathcal{M}(\boldsymbol{y}|\boldsymbol{\Psi}) = \sum_{g=1}^{G} \omega_g f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g),$$

where positive constants $\omega_1, \omega_2, \cdots, \omega_G$ are called weight (or mixing proportions) parameters with this properties that $\sum_{g=1}^{G} \omega_g = 1$ and $\boldsymbol{\Psi} = \big(\boldsymbol{\Theta}_1, \cdots, \boldsymbol{\Theta}_G\big)^\top$ with $\boldsymbol{\Theta}_g = \big(\boldsymbol{\omega}_g, \boldsymbol{\mu}_g, \Sigma_g, \boldsymbol{\lambda}_g\big)^\top$. Herein, $f_{\boldsymbol{Y}}(\boldsymbol{y}, \boldsymbol{\Theta}_g)$ accounts for the density function of random vector $\boldsymbol{Y}$ within g-th component that admits the representation given by

$$\boldsymbol{Y} \overset{d}{=} \boldsymbol{\mu}_g + \sqrt{W}\boldsymbol{\lambda}_g|Z_0| + \sqrt{W}\Sigma_g^{\frac{1}{2}}\boldsymbol{Z}_1,$$

where $\boldsymbol{\mu}_g \in R^d$ is location vector, $\boldsymbol{\lambda}_g \in R^d$ is skewness vector, Σ_g is a positive definite symmetric dispersion matrix for $g = 1, \cdots, G$. Further, W is a positive random variable with mixing density function $f_W(w|\boldsymbol{\theta}_g)$, $Z_0 \sim N(0, 1)$, and $\boldsymbol{Z}_1 \sim N_d\big(\boldsymbol{0}, \Sigma_g\big)$. We note that W, Z_0, and $\boldsymbol{Z}_1$ are mutually independent. For approximating the asymptotic standard error for parameters of the finite mixture model based on observed Fisher information matrix, we use the method of Basford et al. (1997). In fact, the covariance matrix of maximum likelihood (ML) estimator $\hat{\boldsymbol{\Psi}}$, can be approximated by the inverse of the observed information matrix as

$$\sum_{i=1}^{n} \hat{\boldsymbol{h}}_i \hat{\boldsymbol{h}}_i^\top,$$

where

$$\hat{\boldsymbol{h}}_i = \frac{\partial}{\partial \boldsymbol{\Psi}} \log L_i(\hat{\boldsymbol{\Psi}}),$$

and $\log L(\hat{\boldsymbol{\Psi}}) = \sum_{i=1}^{n} \log L_i(\hat{\boldsymbol{\Psi}}) = \sum_{i=1}^{n} \log\Big\{\sum_{g=1}^{G} \widehat{\omega_g} f_{\boldsymbol{Y}}\big(\boldsymbol{y}_i|\widehat{\boldsymbol{\Theta}_g}\big)\Big\}$. Herein $\widehat{\omega}_g$ and $\widehat{\boldsymbol{\Theta}_g}$, for $g = 1, \cdots, G$, denote the ML estimator of ω_g and $\boldsymbol{\Theta}_g$, respectively.

Usage

```
sefm(Y, G, weight, model = "restricted", mu, sigma, lambda, family = "constant",
   skewness = "FALSE", param = NULL, theta = NULL, tick = NULL, h = 0.001, N = 3000,
  level = 0.05, PDF = NULL)
```

Arguments

- `Y` — an $n \times d$ matrix of observations gives the coordinates of the data points.
- `G` — number of components.
- `weight` — a vector of weight parameters (or mixing proportions).
- `model` — it must be `"canonical"`, `"restricted"`, or `"unrestricted"`. By default `model = "restricted"`.
- `mu` — a list of location vectors of `G` components.
- `sigma` — a list of dispersion matrices of `G` components.
- `lambda` — a list of skewness vectors of `G` components. If model is either `"canonical"` or `"unrestricted"`, then skewness vactor must be given in matrix form of appropriate size.
- `family` — name of mixing distribution. By default `family = "constant"` that corresponds to the finite mixture of multivariate normal (or skew normal) distribution. Other candidates for family name are: "bs" (for Birnbaum-Saunders), "burriii" (for Burr type iii), "chisq" (for chi-square), "exp" (for exponential), "f" (for Fisher), "gamma" (for gamma), "gig" (for generalized inverse-Gaussian), "igamma" (for inverse-gamma), "igaussian" (for inverse-Gaussian), "lindley" (for Lindley), "loglog" (for log-logistic), "lognorm" (for log-normal), "lomax" (for Lomax), "pstable" (for positive α-stable), "ptstable" (for polynomially tilted α-stable), "rayleigh" (for Rayleigh), and "weibull" (for Weibull).
- `skewness` — a logical statement. By default `skewness = "FALSE"` which means that a symmetric model is fitted to each component (cluster). If `skewness = "TRUE"`, then a skewed model is fitted to each component.
- `param` — name of the elements of $\boldsymbol{\theta}$ as the parameter vector of mixing distribution with density function $f_W(w|\boldsymbol{\theta})$. By default it is `NULL`.
- `PDF` — mathematical expression for mixing density function $f_W(w|\boldsymbol{\theta})$. By default it is `NULL`.
- `theta` — a list of maximum likelihood estimator for $\boldsymbol{\theta}$ across `G` components. By default it is `NULL`.
- `tick` — a binary vector whose length depends on type of family. The elements of `tick` are either `0` or `1`. If element of `tick` is `0`, then the corresponding element of $\boldsymbol{\theta}$ is not considered in the formula of $f_W(w|\boldsymbol{\theta})$ for computing the required posterior expectations. If element of `tick` is `1`, then the corresponding element of $\boldsymbol{\theta}$ is considered in the formula of $f_W(w|\boldsymbol{\theta})$. For instance, if `family = "gamma"` and either its shape or rate parameter is one, then `tick = c(1)`. This is while, if `family = "gamma"` and both of the shape and rate parameters are in the formula of $f_W(w|\boldsymbol{\theta})$, then `tick = c(1, 1)`. By default `tick = NULL`.
- `h` — a positive small value for computing numerical derivative of $f_W(w|\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$, that is $\partial/\partial\theta f_W(w|\boldsymbol{\theta})$. By default $h = 0.001$.
- `N` — an integer number for approximating the posterior expected values within the E-step of the EM algorithm through the Monte Carlo method. By default $N = 3000$.
- `level` — significance level α for constructing $100(1-\alpha)\%$ confidence interval. By default $\alpha = 0.05$.

Details

Mathematical expressions for density function of mixing distribution $f_W(w|\boldsymbol{\theta})$, are given as follows.

bs

$$f_W(w|\boldsymbol{\theta}) = \frac{\sqrt{\frac{w}{\beta}} + \sqrt{\frac{\beta}{w}}}{2\sqrt{2\pi}\alpha w} \exp\left\{-\frac{1}{2\alpha^2}\left[\frac{w}{\beta} + \frac{\beta}{w} - 2\right]\right\},$$

where $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ and $\beta > 0$ are the first and second parameters of this family, respectively.

burrii

$$f_W(w|\boldsymbol{\theta}) = \alpha\beta w^{-\beta-1}\big(1 + w^{-\beta}\big)^{-\alpha-1},$$

where $w > 0$ and $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ and $\beta > 0$ are the first and second parameters of this family, respectively.

chisq

$$f_W(w|\theta) = \frac{2^{-\frac{\alpha}{2}}}{\Gamma\big(\frac{\alpha}{2}\big)} w^{\frac{\alpha}{2}-1} \exp\Big\{-\frac{w}{2}\Big\},$$

where $w > 0$ and $\theta = \alpha$. Herein $\alpha > 0$ is the degrees of freedom parameter of this family.

exp

$$f_W(w|\theta) = \alpha \exp\big\{-\alpha w\big\},$$

where $w > 0$ and $\theta = \alpha$ where $\alpha > 0$ is the rate parameter of this family.

f

$$f_W(w|\boldsymbol{\theta}) = B^{-1}\Big(\frac{\alpha}{2}, \frac{\beta}{2}\Big)\Big(\frac{\alpha}{\beta}\Big)^{\frac{\alpha}{2}} w^{\frac{\alpha}{2}-1}\Big(1 + \alpha\frac{w}{\beta}\Big)^{-\left(\frac{\alpha+\beta}{2}\right)},$$

where $w > 0$ and $B(.,.)$ denotes the ordinary beta function. Herein $\boldsymbol{\theta} = (\alpha, \beta)^\top$ where $\alpha > 0$ and $\beta > 0$ are the first and second degrees of freedom parameters of this family, respectively.

gamma

$$f_W(w|\boldsymbol{\theta}) = \frac{\beta^\alpha}{\Gamma(\alpha)}\Big(\frac{w}{\beta}\Big)^{\alpha-1} \exp\big\{-\beta w\big\},$$

where $w > 0$ and $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ and $\beta > 0$ are the shape and rate parameters of this family, respectively.

gigaussian

$$f_W(w|\boldsymbol{\theta}) = \frac{1}{2\mathcal{K}_\alpha(\sqrt{\beta\delta})}\Big(\frac{\beta}{\delta}\Big)^{\alpha/2} w^{\alpha-1} \exp\Big\{-\frac{\delta}{2w} - \frac{\beta w}{2}\Big\},$$

where $\mathcal{K}_\alpha(.)$ denotes the modified Bessel function of the third kind with order index α and $\boldsymbol{\theta} = (\alpha, \delta, \beta)^\top$. Herein $-\infty < \alpha < +\infty$, $\delta > 0$, and $\beta > 0$ are the first, second, and third parameters of this family, respectively.

igamma

$$f_W(w|\boldsymbol{\theta}) = \frac{1}{\Gamma(\alpha)}\Big(\frac{w}{\beta}\Big)^{-\alpha-1} \exp\Big\{-\frac{\beta}{w}\Big\},$$

where $w > 0$ and $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters of this family, respectively.

igaussian

$$f_W(w|\boldsymbol{\theta}) = \sqrt{\frac{\beta}{2\pi w^3}} \exp\Bigl\{-\frac{\beta(w-\alpha)^2}{2\alpha^2 w}\Bigr\},$$

where $w > 0$ and $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ is the mean and $\beta > 0$ are the first (mean) and second (shape) parameter of this family, respectively.

lidley

$$f_W(w|\theta) = \frac{\alpha^2}{\alpha+1}(1+w)\exp\bigl\{-\alpha w\bigr\},$$

where $w > 0$ and $\theta = \alpha$ where $\alpha > 0$ is the only parameter of this family.

loglog

$$f_W(w|\boldsymbol{\theta}) = \frac{\alpha}{\beta^\alpha} w^{\alpha-1}\left[\left(\frac{w}{\beta}\right)^\alpha + 1\right]^{-2},$$

where $w > 0$ and $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ and $\beta > 0$ are the shape and scale (median) parameters of this family, respectively.

lognorm

$$f_W(w|\boldsymbol{\theta}) = \bigl(\sqrt{2\pi}\sigma w\bigr)^{-1} \exp\Bigl\{-\frac{1}{2}\left(\frac{\log w - \mu}{\sigma}\right)^2\Bigr\},$$

where $w > 0$ and $\boldsymbol{\theta} = (\mu, \sigma)^\top$. Herein $-\infty < \mu < +\infty$ and $\sigma > 0$ are the first and second parameters of this family, respectively.

lomax

$$f_W(w|\boldsymbol{\theta}) = \alpha\beta\bigl(1+\beta w\bigr)^{-(\alpha+1)},$$

where $w > 0$ and $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ and $\beta > 0$ are the shape and rate parameters of this family, respectively.

rayleigh

$$f_W(w|\theta) = 2\frac{w}{\beta^2}\exp\Bigl\{-\Bigl(\frac{w}{\beta}\Bigr)^2\Bigr\},$$

where $w > 0$ and $\theta = \beta$. Herein $\beta > 0$ is the scale parameter of this family.

weibull

$$f_W(w|\boldsymbol{\theta}) = \frac{\alpha}{\beta}\Bigl(\frac{w}{\beta}\Bigr)^{\alpha-1}\exp\Bigl\{-\Bigl(\frac{w}{\beta}\Bigr)^\alpha\Bigr\},$$

where $w > 0$ and $\boldsymbol{\theta} = (\alpha, \beta)^\top$. Herein $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters of this family, respectively. The density functions of positive α-stable and polynomially tilted α-stable distribution have no closed form and so are not represented here. In what follows, we give four examples. In the first, second, and third examples, we consider three mixture models including: two-component normal, two-component restricted skew t, and two-component restricted skew sub-Gaussian α-stable (SSG) mixture models are fitted to `iris`, `AIS`, and `bankruptcy` data, respectively. In order to approximate the asymptotic standard error of the model parameters, the ML estimators for parameters of skew t and SSG mixture models have been computed through the `R` packages `EMMIXcskew` (developed by Lee and McLachlan (2018) for skew t) and `mixSSG` (developed by Teimouri (2022) for skew sub-Gaussian α-stable). To avoid running package `mixSSG`, we use the ML estimators correspond to `bankruptcy` data provided by Teimouri (2022). The package `mixSSG` is available at https://CRAN.R-project.org/package=mixSSG. In the fourth example, we apply a three-component generalized hyperbolic mixture model to `Wheat` data. The

ML estimators of this mixture model have been obtained using the R package MixGHD available at https://cran.r-project.org/package=MixGHD. Finally, we note that if parameter h is very small (less than 0.001, say), then the approximated observed Fisher information matrix may not be invertible.

Value

A list consists of the maximum likelihood estimator, approximated asymptotic standard error, upper, and lower bounds of $100(1-\alpha)\%$ asymptotic confidence interval for parameters of the finite mixture model.

Author(s)

Mahdi Teimouri

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S. X. Lee and G. J. McLachlan, (2018). EMMIXcskew: An R package for the fitting of a mixture of canonical fundamental skew t-distributions, *Journal of Statistical Software*, 83(3), 1-32, doi: 10.18637/jss.v083.i03.

M. Teimouri, (2022). Finite mixture of skewed sub-Gaussian stable distributions, https://arxiv.org/abs/2205.14067.

C. Tortora, R. P. Browne, A. ElSherbiny, B. C. Franczak, and P. D. McNicholas, (2021). Model-based clustering, classification, and discriminant analysis using the generalized hyperbolic distribution: MixGHD R package. *Journal of Statistical Software*, 98(3), 1-24, doi: 10.18637/jss.v098.i03.

Examples

```
# Example 1: Approximating the asymptotic standard error and 95 percent confidence interval
#           for the parameters of fitted three-component normal mixture model to iris data.
      Y <- as.matrix( iris[, 1:4] )
colnames(Y) <- NULL
rownames(Y) <- NULL
      G <- 3
 weight <- c( 0.334, 0.300, 0.366         )
    mu1 <- c( 5.0060, 3.428, 1.462, 0.246 )
    mu2 <- c( 5.9150, 2.777, 4.204, 1.298 )
    mu3 <- c( 6.5468, 2.949, 5.482, 1.985 )
 sigma1 <- matrix( c( 0.133, 0.109, 0.019, 0.011, 0.109, 0.154, 0.012, 0.010,
              0.019, 0.012, 0.028, 0.005, 0.011, 0.010, 0.005, 0.010 ), nrow = 4 , ncol = 4)
 sigma2 <- matrix( c( 0.225, 0.076, 0.146, 0.043, 0.076, 0.080, 0.073, 0.034,
              0.146, 0.073, 0.166, 0.049, 0.043, 0.034, 0.049, 0.033 ), nrow = 4 , ncol = 4)
 sigma3 <- matrix( c( 0.429, 0.107, 0.334, 0.065, 0.107, 0.115, 0.089, 0.061,
              0.334, 0.089, 0.364, 0.087, 0.065, 0.061, 0.087, 0.086 ), nrow = 4 , ncol = 4)
     mu <- list(    mu1,    mu2,    mu3 )
  sigma <- list( sigma1, sigma2, sigma3 )
  sigma <- list( sigma1, sigma2, sigma3 )
```

```
 lambda <- list( rep(0, 4), rep(0, 4), rep(0, 4) )
   out1 <- sefm( Y, G, weight, model = "restricted", mu, sigma, lambda, family = "constant",
     skewness = "FALSE")
# Example 2: Approximating the asymptotic standard error and 95 percent confidence interval
#           for the parameters of fitted two-component restricted skew t mixture model to
#            AIS data.
      data( AIS )
      Y <- as.matrix( AIS[, 2:3] )
      G <- 2
 weight <- c(  0.5075,  0.4925 )
    mu1 <- c( 19.9827, 17.8882 )
    mu2 <- c( 21.7268,  5.7518 )
 sigma1 <- matrix( c(3.4915, 8.3941, 8.3941, 28.8113 ), nrow = 2, ncol = 2 )
 sigma2 <- matrix( c(2.2979, 0.0622, 0.0622,  0.0120 ), nrow = 2, ncol = 2 )
lambda1 <- ( c( 2.5186, -0.2898 ) )
lambda2 <- ( c( 2.1681,  3.5518 ) )
 theta1 <- c( 68.3088 )
 theta2 <- c(  3.8159 )
     mu <- list(     mu1,     mu2 )
  sigma <- list(  sigma1,  sigma2 )
 lambda <- list( lambda1, lambda2 )
  theta <- list(  theta1,  theta2 )
  param <- c( "nu" )
    PDF <- quote( (nu/2)^(nu/2)*w^(-nu/2 - 1)/gamma(nu/2)*exp( -nu/(w*2) ) )
  tick  <- c( 1, 1 )
   out2 <- sefm( Y, G, weight, model = "restricted", mu, sigma, lambda, family = "igamma",
           skewness = "TRUE", param, theta, tick, h = 0.001, N = 3000, level = 0.05, PDF )
# Example 3: Approximating the asymptotic standard error and 95 percent confidence interval
#            for the parameters of fitted two-component restricted skew sub-Gaussian
#            alpha-stable mixture model to bankruptcy data.
      data( bankruptcy )
      Y <- as.matrix( bankruptcy[, 2:3] );  colnames(Y) <- NULL; rownames(Y) <- NULL
      G <- 2
 weight <- c(  0.553,  0.447 )
    mu1 <- c( -3.649, -0.085 )
    mu2 <- c( 40.635, 19.042 )
 sigma1 <- matrix( c(1427.071, -155.356, -155.356, 180.991 ), nrow = 2, ncol = 2 )
 sigma2 <- matrix( c( 213.938,    9.256,    9.256,  74.639 ), nrow = 2, ncol = 2 )
lambda1 <- c( -41.437, -21.750 )
lambda2 <- c(  -3.666,  -1.964 )
 theta1 <- c( 1.506 )
 theta2 <- c( 1.879 )
     mu <- list(     mu1,     mu2 )
  sigma <- list(  sigma1,  sigma2 )
 lambda <- list( lambda1, lambda2 )
  theta <- list(  theta1,  theta2 )
  param <- c( "alpha" )
  tick  <- c( 1 )
   out3 <- sefm( Y, G, weight, model = "restricted", mu, sigma, lambda, family = "pstable",
            skewness = "TRUE", param, theta, tick, h = 0.01, N = 3000, level = 0.05 )
# Example 4: Approximating the asymptotic standard error and 95 percent confidence interval
#        for the parameters of fitted two-component restricted generalized inverse-Gaussian
#            mixture model to AIS data.
```

```
       data( wheat )
       Y <- as.matrix( wheat[, 1:7] ); colnames(Y) <- NULL; rownames(Y) <- NULL
       G <- 3
  weight <- c( 0.325, 0.341, 0.334 )
     mu1 <- c( 18.8329, 16.2235, 0.9001, 6.0826, 3.8170, 1.6604, 6.0260 )
     mu2 <- c( 11.5607, 13.1160, 0.8446, 5.1873, 2.7685, 4.9884, 5.2203 )
     mu3 <- c( 13.8071, 14.0720, 0.8782, 5.5016, 3.1513, 0.6575, 4.9111 )
 lambda1 <- diag( c( 0.1308, 0.2566,-0.0243, 0.2625,-0.1259, 3.3111, 0.1057) )
 lambda2 <- diag( c( 0.7745, 0.3084, 0.0142, 0.0774, 0.1989,-1.0591,-0.2792) )
 lambda3 <- diag( c( 2.0956, 0.9718, 0.0042, 0.2137, 0.2957, 3.9484, 0.6209) )
  theta1 <- c( -3.3387, 4.2822 )
  theta2 <- c( -3.6299, 4.5249 )
  theta3 <- c( -3.9131, 5.8562 )
  sigma1 <- matrix( c(
  1.2936219, 0.5841467,-0.0027135, 0.2395983, 0.1271193, 0.2263583, 0.2105204,
  0.5841467, 0.2952009,-0.0045937, 0.1345133, 0.0392849, 0.0486487, 0.1222547,
 -0.0027135,-0.0045937, 0.0003672,-0.0033093, 0.0016788, 0.0056345,-0.0033742,
  0.2395983, 0.1345133,-0.0033093, 0.0781141, 0.0069283,-0.0500718, 0.0747912,
  0.1271193, 0.0392849, 0.0016788, 0.0069283, 0.0266365, 0.0955757, 0.0002497,
  0.2263583, 0.0486487, 0.0056345,-0.0500718, 0.0955757, 1.9202036,-0.0455763,
  0.2105204, 0.1222547,-0.0033742, 0.0747912, 0.0002497,-0.0455763, 0.0893237 ), nrow = 7, ncol = 7 )
  sigma2 <- matrix( c(
  0.9969975, 0.4403820, 0.0144607, 0.1139573, 0.1639597,-0.2216050, 0.0499885,
  0.4403820, 0.2360065, 0.0010769, 0.0817149, 0.0525057,-0.0320012, 0.0606147,
  0.0144607, 0.0010769, 0.0008914,-0.0023864, 0.0049263,-0.0122188,-0.0042375,
  0.1139573, 0.0817149,-0.0023864, 0.0416206, 0.0030268, 0.0490919, 0.0407972,
  0.1639597, 0.0525057, 0.0049263, 0.0030268, 0.0379771,-0.0384626,-0.0095661,
 -0.2216050,-0.0320012,-0.0122188, 0.0490919,-0.0384626, 4.0868766, 0.1459766,
  0.0499885, 0.0606147,-0.0042375, 0.0407972,-0.0095661, 0.1459766, 0.0661900 ), nrow = 7, ncol = 7 )
  sigma3 <- matrix( c(
  1.1245716, 0.5527725,-0.0005064, 0.2083688, 0.1190222,-0.4491047, 0.2494994,
  0.5527725, 0.3001219,-0.0036794, 0.1295874, 0.0419470,-0.1926131, 0.1586538,
 -0.0005064,-0.0036794, 0.0004159,-0.0034247, 0.0019652,-0.0026687,-0.0044963,
  0.2083688, 0.1295874,-0.0034247, 0.0715283, 0.0055925,-0.0238820, 0.0867129,
  0.1190222, 0.0419470, 0.0019652, 0.0055925, 0.0243991,-0.0715797, 0.0026836,
 -0.4491047,-0.1926131,-0.0026687,-0.0238820,-0.0715797, 1.5501246,-0.0048728,
  0.2494994, 0.1586538,-0.0044963, 0.0867129, 0.0026836,-0.0048728, 0.1509183 ), nrow = 7, ncol = 7 )
      mu <- list( mu1, mu2, mu3 )
   sigma <- list( sigma1 , sigma2, sigma3 )
  lambda <- list( lambda1, lambda2, lambda3 )
   theta <- list( theta1 , theta2, theta3 )
    tick <- c( 1, 1, 0 )
   param <- c( "a", "b" )
     PDF <- quote( 1/( 2*besselK( b, a ) )*w^(a - 1)*exp( -b/2*(1/w + w) ) )
    out4 <- sefm( Y, G, weight, model = "unrestricted", mu, sigma, lambda, family = "gigaussian",
            skewness = "TRUE", param, theta, tick, h = 0.001, N = 3000, level = 0.05, PDF )
```

`wheat` *wheat data*

Description

These data are about 210 wheat grains belonging to three different varieties (including: Kama, Rosa, and Canadian) on which 7 quantitative variables related to these kernel structures detected by using a soft X-ray visualization technique have been measured. These variables are: `area`, `perimeter`, `compactness`, `length of kernel`, `width of kernel`, `asymmetry coefficient`, `length of kernel groove`, and class label variable `variety`.

Usage

```
data(wheat)
```

Format

A text file with 8 columns.

References

P. Giordani, M. B. Ferraro and F. Martella, (2020). *An Introduction to Clustering with R*, Springer, Singapore.

Examples

```
data(wheat)
```

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