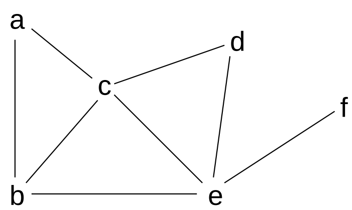
# BCCS 2008/09: Graphical models and complex stochastic systems: Exercises 5

1. Look at slide 36, lecture 6. Verify two things that are claimed there:
   (a) It is in equilibrium - that is, if you try to pass a message from AB to BC or from BC to AB, it does not change the tables.
   (b) The three tables on that slide are indeed marginal distributions for the corresponding variables – which you can check by working these out by hand from the conditional probability tables we started with in slide 30.

2. Look at the finite Hidden Markov model with $T = 3$, and where, for all $t$, both $x_t$ and $y_t$ have just 2 states, 0 (false) and 1 (true). Suppose $p(x_0) = 0.2$ if $x_0 = 0$ and 0.8 if $x_0 = 1$, $p(x_{t+1} = j|x_t = i) = 0, 6, 0.4, 0.1, 0.9$ for $(i, j) = (0, 0), (0, 1), (1, 0), (1, 1)$ respectively, for all $t = 0, 1, \ldots$ and $p(y_t = j|x_t = i) = 0.8, 0.2, 0.2, 0.8$ for $(i, j) = (0, 0), (0, 1), (1, 0), (1, 1)$ respectively, for all $t = 1, 2, \ldots$. For the data $y_t = 0, 1, 0$ for $t = 1, 2, 3$, find $p(x_2|y_1, y_2)$ and $p(x_2|y_1, y_2, y_3)$ using the recursions in section 7.6 of the handouts, and/or by using Hugin (file `hmm.net`, on the web).

3. For the variables in this conditional independence graph:




   decide which of the following conditional independence statements are true. If they are true, state whether they are examples of the global (G), local (L) and/or pairwise (P) Markov property.
   (a) $a \perp\!\!\!\perp e \mid c$
   (b) $a \perp\!\!\!\perp e \mid (b, c)$
   (c) $a \perp\!\!\!\perp d \mid (b, c, e, f)$
   (d) $f \perp\!\!\!\perp (a, b, c) \mid e$
   (e) $d \perp\!\!\!\perp (a, b, f) \mid (c, e)$
   (f) $f \perp\!\!\!\perp a \mid c$
   (g) $(a, d) \perp\!\!\!\perp f \mid (b, c, e)$

4. Of the Markov properties for undirected graphs (lecture 8, slides 19/20), show that G $\Rightarrow$ L (easy) and that F $\Rightarrow$ G. In both cases, an informal verbal explanation is enough, rather than a formal mathematical proof.

5. Consider the autologistic model (lecture 8, slide 25). Assume $p(X)$ has the expression at the top of the slide, and use this to verify that $p(X_i|X_{-i})$ has the stated form.

6. Four non-negative random variables $(x_1, x_2, x_3, x_4)$ have joint distribution $p(x_1, x_2, x_3, x_4) \propto \exp\{-(x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1)\}$. Find the 'full conditional' distribution for $x_1$ given the others, i.e. $p(x_1|x_2, x_3, x_4)$. (What is the name of this distribution?) Noting the symmetry between the variables, hence draw the conditional independence graph for this model, where two variables are joined by an edge if and only if they are not conditionally independent given all other variables. Hence note that the F, L and P Markov properties all hold for this particular model and graph.

7. For the ‘surgical’ example, seen repeatedly, in which $\alpha \sim$ Exponential(1.0), $\beta \sim$ Exponential(1.0), $\theta_i|\alpha, \beta \sim$ Beta($\alpha, \beta$), independently for $i = 1, 2, \ldots, n$ and $x_i|\alpha, \beta, \theta \sim$ Binomial($n_i, \theta_i$), independently for $i = 1, 2, \ldots, n$, find the full conditional distributions for each of the variables $\alpha$, $\beta$ and $\theta_1$ (i.e. the distributions for each of these given the data $(x_i, n_i)$ and all other unknowns).

8. Experiment with Winbugs. The file `WinbugsDemos.zip` on the web has all the files needed for the two examples used in lecture 9, and also the ‘seeds’ example (Winbugs: menu Help | Examples Vol I), together with brief instructions. With a bit of reading of the online manual or viewing of `http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/winbugsthemovie.html` you should be able to try many of the other examples that come with the package, using the graphical interface or by modifying my files.