

Package ‘GAS’

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Type Package

Title Generalized Autoregressive Score Models

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Description Simulate, estimate and forecast using univariate and multivariate GAS models as described in Ardia et al. (2019) <doi:10.18637/jss.v088.i06>.

License GPL-3

BugReports <https://github.com/LeopoldoCatania/GAS/issues>

URL <https://github.com/LeopoldoCatania/GAS>

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Description

The GAS package allows us to simulate, estimate and forecast using univariate and multivariate Generalized Autoregressive Score (GAS) models (also known as Dynamic Conditional Score (DCS) models), see e.g., Creal et. al. (2013) and Harvey (2013) and the website www.gasmodel.com. A detailed implementation of the package functionalities are reported in Ardia et. al. (2018, 2019).

Details

Package: GAS
Type: Package
Date: 2017-11-16
License: GPL (>= 2)

The authors acknowledge Google for financial support via the Google Summer of Code 2016 project "GAS"; see <https://summerofcode.withgoogle.com/archive/2016/projects/4537082387103744/>.

Current limitations:

- The multivariate GAS model for $N > 4$ does not report the exact update for the correlation parameters since the Jacobian of the hyperspherical coordinates transformation needs to be coded for the case $N > 4$. The Jacobian for $N > 4$ is replaced by the identity matrix.

Author(s)

Leopoldo Catania [aut,cre], Kris Boudt [ctb], David Ardia [ctb]

Maintainer: Leopoldo Catania <leopoldo.catania@econ.au.dk>

References

Ardia D, Boudt K and Catania L (2019). "Generalized Autoregressive Score Models in R: The GAS Package." <http://doi.org/10.18637/jss.v088.i06>.

Ardia D, Boudt K and Catania L (2018). "Downside Risk Evaluation with the R Package GAS." <https://journal.r-project.org/archive/2018/RJ-2018-064/index.html>.

Creal D, Koopman SJ, Lucas A (2013). "Generalized Autoregressive Score Models with Applications." *Journal of Applied Econometrics*, 28(5), 777-795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey AC (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press.

BacktestDensity

Backtest a series of one-step ahead density predictions.

Description

The `BacktestDensity()` function accepts an object of the class `uGASRoll`, and returns a list with two elements: (i) the averages Negative Log Score (NLS) and weighted Continuous Ranked Probability Score (wCRPS) introduced by Gneiting and Ranjan (2012), and (ii) their values at each point in time. The wCRPS is evaluated using 5 weight functions, see Details.

Usage

```
BacktestDensity(Roll, lower, upper, K = 1000, a = NULL, b = NULL)
```

Arguments

Roll	an object of the class <code>uGASRoll</code> .
lower	numeric the lower bound used to approximate the wCRSP by Monte Carlo integration as detailed in Gneiting and Ranjan (2012). This coincides with y_l in Equation 16 of Gneiting and Ranjan (2012).
upper	numeric the upper bound used to approximate the wCRSP by Monte Carlo integration as detailed in Gneiting and Ranjan (2012). This coincides with y_u in Equation 16 of Gneiting and Ranjan (2012).
K	numeric integer representing the number of points used to discretize the wCRPS integral. This is I in Equation 16 of Gneiting and Ranjan (2012). By default $K = 1000$.
a	numeric. mean of the normal distribution used in the weight function. By default $a = \text{NULL}$, which means that it is set equal to the empirical mean of the in sample observations.
b	numeric. standard deviation of the normal distribution used in the weight function. By default $b = \text{NULL}$, which means that it is set equal to the empirical standard deviation of the in sample observations.

Details

The average Negative Log Score (NLS) is computed as the negative of the average of the log scores evaluated during the out-of-sample period. The average weighted Continuous Ranked Probability Score (wCRPS) is computed as the average of the wCRPS evaluated during the out-of-sample period, see Gneiting and Ranjan (2012).

The wCRPS is evaluated using Equation 16 of Gneiting and Ranjan (2012). The weights functions implemented are:

- $w(z) = 1$: Uniform,
- $w(z) = \phi_{a,b}(z)$: Center,
- $w(z) = 1 - \phi_{a,b}(z)$: Tails,
- $w(z) = \Phi_{a,b}(z)$: Right tail,
- $w(z) = 1 - \Phi_{a,b}(z)$: Left tail,

where $\phi_{a,b}(z)$ and $\Phi_{a,b}(z)$ are the pdf and cdf of a Gaussian distribution with mean a and standard deviation b , respectively. The label "Uniform" represents the case where equal emphasis is given to all the parts of the distribution.

Value

A list with elements: average, series. The element "average" is a named vector with the averages NLS and wCRSP. The element "series" is a list: the first element, LS, contains the out-of-sample Log Score (not with the negative sign), the second element, WCRPS, contains a matrix with the wCRPS series. The columns of this matrix are named: "uniform", "center", "tails", "tail_r", "tail_l", which are associated with the wCRSP with emphasis on: Uniform, Center, Tails, Right tail and Left tail, respectively.

Author(s)

Leopoldo Catania

References

Gneiting T, Ranjan R (2011). "Comparing Density Forecasts using Threshold -and Quantile-Weighted Scoring Rules." *Journal of Business & Economic Statistics*, 29(3), 411-422. doi: [10.1198/jbes.2010.08110](https://doi.org/10.1198/jbes.2010.08110).

Examples

```
## Not run:
data("cpichg")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = TRUE, scale = TRUE,
                                   shape = FALSE))

Roll = UniGASRoll(cpichg, GASSpec, ForecastLength = 50,
                 RefitEvery = 10, RefitWindow = c("moving"))

BackTest = BacktestDensity(Roll, lower = -100, upper = 100)

BackTest$average

## End(Not run)
```

BacktestVaR

Backtest Value at Risk (VaR)

Description

This function implements several backtesting procedures for the Value at Risk (VaR). These are: (i) The statistical tests of Kupiec (1995), Christoffesen (1998) and Engle and Manganelli (2004), (ii) The tick loss function detailed in Gonzalez-Rivera et al. (2004), the mean and max absolute loss used by McAleer and Da Veiga (2008) and the actual over expected exceedance ratio.

Usage

```
BacktestVaR(data, VaR, alpha, Lags = 4)
```

Arguments

data	numeric Vector of observations.
VaR	numeric Vector containing the VaR series.
alpha	numeric The VaR confidence level.
Lags	numeric Lags used in the Dynamic Quantile test of Engle and Manganelli (2004).

Details

This function implements several backtesting procedure for the Value at Risk. The implemented statistical tests are:

- LRuc The unconditional coverage test of Kupiec (1995).
- LRcc The conditional coverage test of Christoffesen (1998).
- DQ The Dynamic Quantile test of Engle and Manganelli (2004).

The implemented VaR backtesting quantities are:

- AD mean and maximum absolute deviation between the observations and the quantiles as in McAleer and Da Veiga (2008).
- Loss Average quantile loss and quantile loss series as in Gonzalez-Rivera et al. (2004).
- AE Actual over Expected exceedance ratio.

Value

A list with elements: LRuc, LRcc, DQ, AD, AE.

Author(s)

Leopoldo Catania

References

Christoffersen PF (1998). "Evaluating Interval Rorecasts." *International Economic Review*, 39(4), 841-862. doi: [10.2307/2527341](https://doi.org/10.2307/2527341).

Engle RF and Manganelli S. (2004). "CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles." *Journal of Business & Economic Statistics*, 22(4), 367-381. doi: [10.1198/073500104000000370](https://doi.org/10.1198/073500104000000370).

Gonzalez-Rivera G, Lee TH, and Mishra, S (2004). "Forecasting Volatility: A Reality Check Based on Option Pricing, Utility Function, Value-at-Risk, and Predictive Likelihood." *International Journal of Forecasting*, 20(4), 629-645. doi: [10.1016/j.ijforecast.2003.10.003](https://doi.org/10.1016/j.ijforecast.2003.10.003).

Kupiec PH (1995). "Techniques for Verifying the Accuracy of Risk Measurement Models." *The Journal of Derivatives*, 3(2), 73-84. doi: [10.3905/jod.1995.407942](https://doi.org/10.3905/jod.1995.407942)

McAleer M and Da Veiga B (2008). "Forecasting Value-at-Risk with a Parsimonious Portfolio Spillover GARCH (PS-GARCH) Model." *Journal of Forecasting*, 27(1), 1-19. doi: [10.1002/for.1049](https://doi.org/10.1002/for.1049).

Examples

```
data("StockIndices")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = FALSE, scale = TRUE,
                                   shape = FALSE))

FTSEMIB = StockIndices[, "FTSEMIB"]

InSampleData = FTSEMIB[1:1500]
OutSampleData = FTSEMIB[1501:2404]

Fit = UniGASFit(GASSpec, InSampleData)
Forecast = UniGASFor(Fit, Roll = TRUE, out = OutSampleData)

alpha = 0.05

VaR = quantile(Forecast, alpha)

BackTest = BacktestVaR(OutSampleData, VaR, alpha)
```

ConfidenceBands

Build confidence bands for the filtered parameters

Description

Build confidence bands for the filtered parameters sampling the coefficients from the asymptotic distribution as in Blasques et al. (2016).

Usage

```
ConfidenceBands(object, B = 10000, probs = c(0.01,0.1,0.9,0.99), ...)
```

Arguments

object	An object of the class <code>uGASFit</code> or <code>mGASFit</code>
B	numeric Number of draws from the asymptotic distributions.
probs	numeric Quantiles to returns.
...	Additional arguments.

Details

This function implements the "In-Sample Simulation-Based Bands" Section 3.3 of Blasques et al. (2016).

Value

An object of the class array of dimension $(T+1) \times B \times K$, where T is the length of the time series, K is the number of parameters and B the number of draws. The first slice reports the estimated filtered parameters. The one step ahead prediction is also reported, this why $T+1$.

Author(s)

Leopoldo Catania

References

Blasques F, Koopman SJ, Lasak K, and Lucas, A (2016). "In-sample Confidence Bands and Out-of-Sample Forecast Bands for Time-Varying Parameters in Observation-Driven Models." *International Journal of Forecasting*, 32(3), 875-887. doi: [10.1016/j.ijforecast.2016.04.002](https://doi.org/10.1016/j.ijforecast.2016.04.002).

Examples

```
## Not run:

# show the information of all the supported distributions
library("GAS")

data("cpichg")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = TRUE, scale = TRUE,
                                   shape = FALSE))

Fit = UniGASFit(GASSpec, cpichg)

Bands = ConfidenceBands(Fit)

## End(Not run)
```

cpichg

Data: Quarterly logarithmic change in percentage points of the Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL) from 1947-04-01 to 2016-05-01

Description

Quarterly logarithmic change in percentage points of the Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL) from 1947-04-01 to 2016-05-01 available at <https://fred.stlouisfed.org/series/CPIAUCSL>.

Usage

```
data("cpichg")
```

Format

A `xts` object containing 276 observations from 1947-04-01 to 2016-05-01.

References

US. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items [CPI-AUCSL], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/CPIAUCSL>, June 24, 2016.

 DistInfo

Information for the supported distributions

Description

Print the information regarding distributions supported in the GAS package.

Usage

```
DistInfo(DistLabel = NULL, N = 2, FULL = TRUE)
```

Arguments

DistLabel	character indicating the label of the conditional distribution. if DistLabel = NULL all the supported distributions are printed. Default DistLabel = NULL. Run DistLabels() to see the labels of the currently implemented distributions.
N	numeric Indicating the number of asset if DistLabel link to a multivariate distribution. Default N = 2.
FULL	logical If TRUE the function prints all the the information. Default FULL = TRUE

Details

The information are printed in the console.

Author(s)

Leopoldo Catania

Examples

```
# show the information of all the supported distributions
library("GAS")

DistInfo()
```

distributions *Distributions of the GAS package*

Description

Density, distribution function, quantile function, random generator, moments, scores and information matrix of univariate and multivariate distributions of the GAS package.

Usage

```
ddist_Uni(y, Theta, Dist, log = FALSE)
pdist_Uni(q, Theta, Dist)
qdist_Uni(p, Theta, Dist)
rdist_Uni(Theta, Dist)
mdist_Uni(Theta, Dist)
Score_Uni(y, Theta, Dist)
IM_Uni(Theta, Dist)

ddist_Multi(y, Theta, Dist, log = FALSE)
rdist_Multi(Theta, N, Dist)
Score_Multi(y, Theta, Dist)
```

Arguments

<code>y, q</code>	numeric Scalar quantile. For <code>Score_Multi</code> and <code>ddist_Multi</code> , <code>y</code> is a numeric vector.
<code>p</code>	numeric Scalar probability.
<code>Theta</code>	numeric Vector of distribution parameters. The order of the parameters is generally: location, scale, skewness, shape, shape2. When the distribution defined by <code>Dist</code> does not have, say, the shape parameter, this should be simply omitted. See also DistInfo for specific distributions.
<code>Dist</code>	character Label of the conditional distribution, see DistInfo .
<code>log</code>	logical If TRUE, the density value $p(y)$ is given as $\log(p(y))$. Dy Default <code>log = FALSE</code> .
<code>N</code>	numeric Integer. cross-sectional dimension for the multivariate case.

Details

The function `mdist_Uni` returns a vector with four elements: mean, variance, skewness and kurtosis coefficients. The functions `Score_Uni` and `IM_Uni` returns the score and the Fisher information matrix for univariate distributions. The function `Score_Multi` returns the score for multivariate distributions. See [DistInfo](#) for the lists of supported distributions. These functions are not vectorized. `ddist_Uni` and `ddist_Multi` give the density, `pdist_Uni` gives the distribution function, `qdist_Uni` gives the quantile function, and `rdist_Uni` and `rdist_Multi` generate random deviates.

Value

1. numeric scalar for: `ddist_Uni`, `pdist_Uni`, `qdist_Uni`, `rdist_Uni`,
2. numeric vector for: `Score_Uni`, `Score_Multi` and `rdist_Multi`,
3. matrix for `IM_Uni`.

Author(s)

Leopoldo Catania

Examples

```
# Skew Student-t distribution

# log density
Theta = c("location" = 0, "scales" = 1, "skewness" = 1.2, "shape" = 7)

ddist_Uni(y = 0.5, Theta, "sstd", TRUE)

# probability
pdist_Uni(q = -1.69, Theta, "sstd")

#quantile
qdist_Uni(p = 0.05, Theta, "sstd")

#random generator
rdist_Uni(Theta, "sstd")

#moments
mdist_Uni(Theta, "sstd")
```

dji30ret

data: Dow Jones 30 Constituents Closing Value Log Return in percentage points

Description

This dataset is taken from the `rugarch` package of Ghalanos (2015). Returns are in percentage points.

Dow Jones 30 Constituents closing value log returns from 1987-03-16 to 2009-02-03 from Yahoo Finance. Note that AIG was replaced by KFT (Kraft Foods) on September 22, 2008. This is not reflected in this data set as that would bring the starting date of the data to 2001.

Usage

```
data("dji30ret")
```

Format

A data.frame containing 5,521x30 observations.

Source

Yahoo Finance

References

Ghalanos A (2015). "rugarch: Univariate GARCH models." <https://cran.r-project.org/package=rugarch>.

fn.optim

A wrapper to the `optim` function.

Description

This function is a wrapper to the standard `optim` optimizer with `method = "BFGS"`.

Usage

```
fn.optim(par0, data, GASSpec, FUN)
```

Arguments

<code>par0</code>	numeric vector of named model coefficients.
<code>data</code>	numeric vector or matrix of data.
<code>GASSpec</code>	An object of the class <code>uGASSpec</code> or <code>mGASSpec</code> , created via the <code>UniGASSpec</code> and <code>MultiGASSpec</code> functions.
<code>FUN</code>	A function to optimize.

Details

The following control parameters are used for control:

- `trace = 0`
- `abstol = 1e-8`

See the documentation of [optim](#).

Value

It returns a named list with four elements: i) `pars`: a numeric vector where the estimated parameters are stored, ii) `value`: a numeric containing the value of the negative log likelihood evaluated at its minimum, iii) `hessian`, a numeric matrix containing the Hessian matrix evaluated at the minimum of the negative log likelihood, iv) `convergence` a numeric element indicating the convergence results of [optim](#).

Author(s)

Leopoldo Catania

See Also

help(optim)

fn.solnp	<i>A wrapper to the solnp function of the Rsolnp package of Ghalanos and Theussl (2016).</i>
----------	--

Description

This function is a wrapper to the [solnp](#) function of the Rsolnp package of Ghalanos and Theussl (2016).

Usage

```
fn.solnp(par0, data, GASSpec, FUN)
```

Arguments

par0	numeric vector of named model coefficients.
data	numeric vector or matrix of data.
GASSpec	An object of the class uGASSpec or mGASSpec , created via the UniGASSpec and MultiGASSpec functions.
FUN	A function to optimize.

Details

The following control parameters are used: trace = 0, rho = 1, outer.iter = 400, inner.iter = 1800, delta = 1e-08, tol = 1e-08. See the documentation of [solnp](#).

Value

It returns a named list with four elements: i) pars: a numeric vector where the estimated parameters are stored, ii) value: a numeric containing the value of the negative log likelihood evaluated at its minimum, iii) hessian, a numeric matrix containing the Hessian matrix evaluated at the minimum of the negative log likelihood, and iv) convergence a numeric element indicating the convergence results of [solnp](#).

Author(s)

Leopoldo Catania

References

Alexios Ghalanos and Stefan Theussl (2015). "Rsolnp: General Non-linear Optimization Using Augmented Lagrange Multiplier Method". R package version 1.16.

See Also

help(solnp)

FZLoss	<i>Fissler and Ziegel (2016) (FZ) joint loss function for Value at Risk and Expected Shortfall.</i>
--------	---

Description

This function implements Fissler and Ziegel (2016) (FZ) joint loss function for Value at Risk and Expected Shortfall.

Usage

```
FZLoss(data, VaR, ES, alpha)
```

Arguments

data	numeric Vector of observations.
VaR	numeric Vector containing the VaR series.
ES	numeric Vector containing the ES series.
alpha	numeric The VaR and ES confidence level.

Details

This function implements Fissler and Ziegel (2016) (FZ) joint loss function for Value at Risk and Expected Shortfall. The parameterization used is that of Patton et al. (2017) and is given by:

$$\frac{1}{\alpha ES_t^\alpha} I_t^\alpha(y_t - VaR_t^\alpha) + \frac{VaR_t^\alpha}{ES_t^\alpha} + \log -ES_t^\alpha - 1.$$

See also Fissler et al. (2015).

Value

A numeric vector containing the joining VaR and ES loss values.

Author(s)

Leopoldo Catania

References

Fissler, T., Ziegel, J.F., (2016). "Higher order elicibility and Osband's principle." *The Annals of Statistics* 44, 1680-1707. doi: [10.1214/16AOS1439](https://doi.org/10.1214/16AOS1439).

Fissler, T., Ziegel, J.F., Tilmann, G. (2015). "Expected Shortfall is jointly elicitable with Value at Risk - Implications for backtesting." arXiv preprint arXiv:1507.00244.

Patton, A. J., Ziegel, J.F., Chen, R. (2017). "Dynamic semiparametric models for expected shortfall (and Value-at-Risk)." arXiv preprint arXiv:1707.05108.

Examples

```
data("StockIndices")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = FALSE, scale = TRUE,
                                   shape = FALSE))

FTSEMIB = StockIndices[, "FTSEMIB"]

InSampleData = FTSEMIB[1:1500]
OutSampleData = FTSEMIB[1501:2404]

Fit = UniGASFit(GASSpec, InSampleData)

Forecast = UniGASFor(Fit, Roll = TRUE, out = OutSampleData)

alpha = 0.05

vVaR = quantile(Forecast, alpha)
vES = ES(Forecast, alpha)

FZ = FZLoss(OutSampleData, vVaR, vES, alpha)
```

Goals

data: Goals scored by England against Scotland in international football matches.

Description

Number of goals scored by England against Scotland in international football matches. This is a 116 x 2 [zoo](#) object spanning the period 1872-1987 with a yearly frequency. The first column reports the number of goals scored by England against Scotland. The second column is a dummy variable equal 1 for matches played in England. This data set is taken from the Harvey (1989) pg 524.

Usage

```
data("Goals")
```

Format

A `zoo` object containing 116 x 2 observations.

Source

Harvey (1989) pg. 524

References

Harvey, A. C. (1990). Forecasting, structural time series models and the Kalman filter. Cambridge university press. <https://cran.r-project.org/package=rugarch>.

mGASFit

Class for the Multivariate GAS fitted object

Description

Class for the multivariate GAS fitted object.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

Data: Object of class `list`. Contains the user's data.

Estimates: Object of class `list`. Contains: `lParList` list of estimated parameters, optimiser object delivered from the optimization function, `StaticFit` ML estimates for the constant model, Inference inferential results for the estimated parameters.

GASDyn: Object of class `list`. Contains: the series of filtered dynamic (`GASDyn$mTheta`) for the time-varying parameters, the series of scaled scores (`GASDyn$mInnovation`), the series of unrestricted filtered parameters (`GASDyn$mTheta_tilde`), the series of log densities (`GASDyn$vLLK`), the log likelihood evaluated at its optimum value (`GASDyn$dLLK`)

ModelInfo: Object of class `list`. Contains information about the GAS specification:

- `Spec` Object of the class `uGASSpec` containing the GAS specification.
- `iT` numeric Number of observation.
- `elapsedTime` Numeric elapsed time in seconds.

Methods

- `show signature(object = 'mGASFit')`: print object information.
- `summary signature(object = 'mGASFit')`: Show summary.
- `plot signature(x='mGASFit', y='missing')`: Plot filtered dynamic and other estimated quantities.
- `getFilteredParameters signature(object = "mGASFit")`: Extract filtered parameters.
- `getObs signature(object = "mGASFit")`: Extract original observations.
- `coef signature(object = 'uGASFit')`: Returns a named vector of estimated coefficients. Also accepts the additional logical argument `do.list`. If `do.list = TRUE`, estimated coefficients are organized in a list with arguments: `vKappa` the intercept vector, `mA` the A system matrix, `mB` the B system matrix. By default, `do.list = FALSE`.
- `getMoments signature(object = "mGASFit")`: Extract conditional moments.
- `residuals signature(object = 'mGASFit')`: Extract the residuals. Also accepts the additional logical argument `standardize`. If `standardize = TRUE`, residuals are standardized by cholesky of the filtered covariance matrix. By default `standardize = FALSE`.
- `convergence signature(object = 'mGASFit')`: Extract convergence information.

Author(s)

Leopoldo Catania

mGASFor

Class for the Multivariate GAS Forecast object

Description

Class for the multivariate GAS forecast object.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

Forecast: Object of class `list`. Contains forecasts:

- **PointForecast:** `matrix` with parameters forecasts.
- **Moments:** `list` with centered moments forecasts. The first element contains a `matrix` with the predicted conditional means. The second element contains an array with the predicted conditional covariances.
- **vLS:** numeric Log Score (Predictive Log Likelihood).

Bands: array with confidence bands parameters forecasts. Available only if `Roll = TRUE`.

Draws: If `ReturnsDraws = TRUE` it is a `iH x iB matrix` of draws from the predictive distribution.

Info: `list` with forecast information.

Data: `list` with original data.

Methods

- `show signature(object = "uGASFor")`: Show summary.
- `plot signature(x='uGASFor',y='missing')`: Plot forecasted quantities.
- `getForecast signature(object = "uGASFor")`: Extract parameters forecast.
- `getObs signature(object = "uGASFor")`: Extract original observations.
- `getMoments signature(object = "uGASFor")`: Extract moments forecasts.
- `LogScore signature(object = "uGASFor")`: Extract Log Scores.

Author(s)

Leopoldo Catania

mGASRoll

Class for the Multivariate GAS Rolling object

Description

Class for the multivariate GAS rolling object.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

Forecast: Object of class `list`. Contains forecasts:

- **PointForecast:** matrix with parameters forecasts.
- **Moments:** list with centered moments forecasts. The first element contains a matrix with the predicted conditional means. The second element contains an array with the predicted conditional covariances.
- **vLS:** numeric Log Score (Predictive Log Likelihood).

Info: list with forecast information.

Data: list with original data.

Methods

- `show signature(object = 'mGASRoll')`: Show summary.
- `plot signature(x = 'mGASRoll',y = 'missing')`: Plot forecasted quantities.
- `getForecast signature(object = 'mGASRoll')`: Extract parameters forecast.
- `getObs signature(object = 'mGASRoll')`: Extract original observations.
- `getMoments signature(object = 'mGASRoll')`: Extract moments forecasts.
- `LogScore signature(object = 'mGASRoll')`: Extract Log Scores.

- `residuals` signature(object = 'mGASRoll'): Extract the forecast errors. Also accepts the additional logical argument `standardize`. If `standardize = TRUE`, forecast errors are standardized by cholesky of the forecast covariance matrix. By default `standardize = FALSE`.
- `coef` signature(object = 'mGASFit'): Returns a matrix of estimated coefficients. Each row of the matrix corresponds to a refit of the model during the forecast period according to the `RefitEvery` argument provided in the [MultiGASRoll](#) function. Also accepts the additional logical argument `do.list`. If `do.list = TRUE`, estimated coefficients are organized in a list of lists according to the `RefitEvery` argument provided in the [MultiGASRoll](#) function. Each list is populated by three arguments: `vKappa` the intercept vector, `mA` the A system matrix, `mB` the B system matrix. By default, `do.list = FALSE`.

Author(s)

Leopoldo Catania

mGASSim

Class for Multivariate GAS Simulation

Description

Class for multivariate GAS model simulation.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

ModelInfo: Object of class `list`. Contains information about the multivariate GAS specification:

- `iT`: numeric Time length of simulated observations.
- `iN`: numeric Cross sectional dimension.
- `iK`: numeric number of (possibly) time-varying parameters implied by the distributional assumption.
- `vKappa` numeric vector of unconditional level for the reparametrized vector of parameters.
- `mA` matrix of coefficients of dimension `iK x iK` that premultiply the conditional score in the GAS updating recursion.
- `mB` matrix of autoregressive coefficients of dimension `iK x iK`.
- `Dist` character label of the conditional distribution, see [DistInfo](#)
- `ScalingType` character representing the scaling mechanism for the conditional score, see [DistInfo](#)

GASDyn: Object of class `list`. Contains: the series of simulated parameters (`GASDyn$mTheta`), the series of scaled scores (`GASDyn$mInnovation`), the series of unrestricted simulated parameters (`GASDyn$mTheta_tilde`), the series of log densities (`GASDyn$vLLK`), the log likelihood evaluated at its optimum value (`GASDyn$dLLK`)

Data: Object of class `matrix`. Matrix of dimension `iN x iT` of simulated data

Methods

- `show signature(object = 'mGASSim')`: Show summary.
- `plot signature(x = 'mGASSim', y = 'missing')`: Plot simulated data and parameters.
- `getFilteredParameters signature(object = 'mGASSim')`: Extract simulated parameters.
- `getObs signature(object = 'mGASSim')`: Extract simulated observations
- `coef signature(object = 'mGASSim')`: Extract delivered coefficients
- `getMoments signature(object = 'uGASFor')`: Extract simulated moments.

Author(s)

Leopoldo Catania

mGASSpec

Class for the Multivariate GAS model specification

Description

Class for the Multivariate GAS model specification.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

Spec: Object of class `list`. Contains information about the multivariate GAS specification:

- `Dist`: character Containing the conditional distribution assumption.
- `ScalingType`: character indicating the scaling mechanism for the conditional score.
- `GASPar`: list with elements: location, scale, correlation, shape.
- `ScalarParameters`: logical indicates if the parameters of the locations, scales and correlations dynamic have to be scalars or a diagonal matrices.

Methods

- `show signature(object = 'mGASSpec')`: Show summary.

Author(s)

Leopoldo Catania

MultiGASFit *Estimate multivariate GAS models*

Description

Estimate multivariate GAS models by Maximum Likelihood.

Usage

```
MultiGASFit(GASSpec, data, fn.optimizer = fn.optim, Compute.SE = TRUE)
```

Arguments

GASSpec	An object of the class <code>mGASSpec</code> created using the function <code>MultiGASSpec</code>
data	<code>matrix</code> (or something coercible to that using <code>as.matrix()</code>) of dimension TxN containing the multivariate time series of observations. It can also be an object of the class <code>ts</code> , <code>xts</code> or <code>zoo</code> .
fn.optimizer	function. This is a generic optimization function that can be provided by the user. By default <code>fn.optimizer = fn.optim</code> where <code>fn.optim</code> is a wrapper to the <code>optim</code> function. See Details for user defined optimization routines.
Compute.SE	logical. Should asymptotic Standard Errors be computed? By default <code>Compute.SE = TRUE</code>

Details

Maximum Likelihood estimation of GAS models is an on-going research topic. General results are reported by Blasques et al. (2014b), Blasques et al. (2014a) and Harvey (2013), while results for specific models have been derived by Blasques et al. (2014c) and Andres (2014).

Starting values for the optimizer are chosen in the following way: (i) estimate the static version of the model (i.e., with $A = 0$ and $B = 0$) and set the initial value of the intercept parameter accordingly, and (ii) perform a grid search for the coefficients contained in A and B. Further technical details are presented in Section 3.2 of Ardia et. al. (2016a).

The user is free to employ his/her own optimization routine via the `fn.optimizer` argument. `fn.optimizer` accepts a function object. The user provided optimizer has to satisfy strict requirements. The arguments of the `fn.optimizer` are : i) `par0` a vector of starting values, ii) `data` the data provided, iii) `GASSpec` an object of the class `uGASSpec`, and iv) `FUN` the likelihood function. The output of `fn.optimizer` has to be an object of the class `list` with four named elements: i) `pars`: a numeric vector where the estimated parameters are stored, ii) `value`: a numeric containing the value of the negative log likelihood evaluated at its minimum, iii) `hessian`, a numeric matrix containing the Hessian matrix evaluated at the minimum of the negative log likelihood, this is used for inferential purposes, and iv) `convergence` a numeric variable reporting information about the convergence of the optimization. This quantity is printed by the `show()` and `summary()` methods. `convergence = 0` has to indicate successful completion.

The user is allowed to not include the last two elements of the output of the `fn.optimizer` function, that is, the values `hessian = NULL` and `convergence = NULL` are admissible. In the case of

hessian = NULL, the Hessian matrix is evaluated numerically using the `hessian` function in the `numDeriv` package of Gilbert and Varadhan (2016). If the provided hessian is not positive definite, a try with the hessian evaluation used by the BFGS quasi-Newton implementation in the function `optim` is made.

By default, the `optim` optimizer with `method = "BFGS"` is employed.

Value

An object of the class `mGASFit`.

Author(s)

Leopoldo Catania

References

Ardia D, Boudt K and Catania L (2016a). "Generalized Autoregressive Score Models in R: The GAS Package." <http://ssrn.com/abstract=2825380>.

Blasques F, Koopman SJ, Lucas A (2014a). "Maximum Likelihood Estimation for Correctly Specified Generalized Autoregressive Score Models: Feedback Effects, Contraction Conditions and Asymptotic Properties." techreport TI 14-074/III, Tinbergen Institute. <http://www.tinbergen.nl/discussionpaper/?paper=2332>.

Blasques F, Koopman SJ, Lucas A (2014b). "Maximum Likelihood Estimation for Generalized Autoregressive Score Models." techreport TI 2014-029/III, Tinbergen Institute. <http://www.tinbergen.nl/discussionpaper/?paper=2286>.

Blasques F, Koopman SJ, Lucas A, Schaumburg J (2014c). "Spillover Dynamics for Systemic Risk Measurement using Spatial Financial Time Series Models." techreport TI 2014-103/III, Tinbergen Institute. <http://www.tinbergen.nl/discussionpaper/?paper=2369>.

Creal D, Koopman SJ, Lucas A (2013). "Generalized Autoregressive Score Models with Applications." *Journal of Applied Econometrics*, 28(5), 777-795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Ghalanos A, Theussl S (2016). "Rsolnp: General Non-Linear Optimization using Augmented Lagrange Multiplier Method." <https://cran.r-project.org/package=Rsolnp>.

Gilbert P, Varadhan R (2016). `numDeriv`: Accurate Numerical Derivatives. R package 2016.8-1, <https://CRAN.R-project.org/package=numDeriv>.

Harvey AC (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press.

Ye Y (1988). *Interior Algorithms for Linear, Quadratic, and Linearly Constrained Convex Programming*. Ph.D. thesis, Stanford University.

Examples

```
## Not run:
# Specify an GAS model with multivariate Student-t
# conditional distribution and time-varying scales and correlations

library("GAS")

data("StockIndices")

GASSpec = MultiGASSpec(Dist = "mvt", ScalingType = "Identity",
                      GASPar = list(scale = TRUE, correlation = TRUE))

Fit = MultiGASFit(GASSpec, StockIndices)

Fit

## End(Not run)
```

MultiGASFor

Forecast with multivariate GAS models

Description

Forecast with multivariate GAS models. One-step ahead prediction of the conditional density is available in closed form. Multistep ahead prediction are performed by simulation as detailed in Blasques et al. (2016).

Usage

```
MultiGASFor(mGASFit, H = NULL, Roll = FALSE, out = NULL, B = 10000,
            Bands = c(0.1, 0.15, 0.85, 0.9), ReturnDraws = FALSE)
```

Arguments

mGASFit	An object of the class mGASFit created using the function MultiGASFit
H	numeric Forecast horizon. Ignored if Roll = TRUE
Roll	logical Forecast should be made using a rolling procedure ? Note that if Roll = TRUE, then out has to be specified.
out	matrix of out of sample observation of dimension H x N for rolling forecast. N refers to the cross sectional dimension.
B	numeric Number of draws from the iH-step ahead distribution if Roll = FALSE.
Bands	numeric Vector of probabilities representing the confidence band levels for multistep ahead parameters forecasts. Only if Roll = FALSE.
ReturnDraws	logical Return the draws from the multistep ahead predictive distribution when Roll = FALSE ?

Value

An object of the class `mGASFor`

Author(s)

Leopoldo Catania

References

Blasques F, Koopman SJ, Lasak K, and Lucas, A (2016). "In-sample Confidence Bands and Out-of-Sample Forecast Bands for Time-Varying Parameters in Observation-Driven Models." *International Journal of Forecasting*, 32(3), 875-887. doi: [10.1016/j.ijforecast.2016.04.002](https://doi.org/10.1016/j.ijforecast.2016.04.002).

Examples

```
## Not run:
# Specify a GAS model with multivariate Student-t conditional
# distribution and time-varying scales and correlations.

# Stock returns forecast

set.seed(123)

data("StockIndices")

mY = StockIndices[, 1:2]

# Specification mvt
GASSpec = MultiGASSpec(Dist = "mvt", ScalingType = "Identity",
                      GASPar = list(location = FALSE, scale = TRUE,
                                     correlation = TRUE, shape = FALSE))

# Perform H-step ahead forecast with confidence bands

# Estimation
Fit = MultiGASFit(GASSpec, mY)

# Forecast
Forecast = MultiGASFor(Fit, H = 50)

Forecast

# Perform 1-Step ahead rolling forecast

InSampleData = mY[1:1000, ]
OutSampleData = mY[1001:2404, ]

# Estimation
Fit = MultiGASFit(GASSpec, InSampleData)

Forecast = MultiGASFor(Fit, Roll = TRUE, out = OutSampleData)
```



```
Forecast
## End(Not run)
```

MultiGASRoll

Rolling forecast with multivariate GAS models

Description

One-step ahead rolling forecasts with model re-estimation. The function also reports several quantity for backtesting for point and density forecasts.

Usage

```
MultiGASRoll(data, GASSpec, ForecastLength = 500, Nstart = NULL,
             RefitEvery = 23, RefitWindow = c("moving", "recursive"),
             cluster = NULL, Compute.SE = FALSE, ...)
```

Arguments

<code>data</code>	matrix of dimension $(T + \text{ForecastLength}) \times N$ containing the time series of observations.
<code>GASSpec</code>	An object of the class <code>mGASSpec</code> created using the function <code>MultiGASSpec</code>
<code>ForecastLength</code>	numeric Length of the out of sample
<code>Nstart</code>	numeric Period when perform the first forecast. Ignored if <code>ForecastLength</code> is supplied.
<code>RefitEvery</code>	numeric Number of periods before model coefficients re-estimation.
<code>RefitWindow</code>	character Type of window. If <code>RefitWindow = "recursive"</code> all the observations are used when the model is re-estimated. If <code>RefitWindow = "moving"</code> old observations are eliminated.
<code>cluster</code>	A <code>cluster</code> object created calling using the <code>parallel</code> package. If supplied parallel processing is used to speed up the computations.
<code>Compute.SE</code>	logical. Should asymptotic Standard Errors be computed? By default <code>Compute.SE = FALSE</code>
<code>...</code>	Additional arguments for <code>MultiGASFit</code>

Value

An object of the class `mGASRoll`

Author(s)

Leopoldo Catania

Examples

```

## Not run:
# Specify a GAS model with Multivariate Student-t conditional
# distribution and time-varying scale and correlation parameters

# stock returns Forecast

data("StockIndices")

mY = StockIndices[, 1:2]

# Specification mvt
GASSpec = MultiGASSpec(Dist = "mvt", ScalingType = "Identity",
                      GASPar = list(location = FALSE, scale = TRUE,
                                     correlation = TRUE, shape = FALSE))

# Perform 1-step ahead rolling forecast with refit
library(parallel)

Roll = MultiGASRoll(mY, GASSpec, ForecastLength = 250,
                   RefitEvery = 100, RefitWindow = c("moving"))

Roll

## End(Not run)

```

MultiGASSim

Simulate multivariate GAS processes

Description

Simulate multivariate GAS processes.

Usage

```

MultiGASSim(fit = NULL, T.sim = 1000, N = NULL,
            kappa = NULL, A = NULL, B = NULL, Dist = NULL, ScalingType = NULL)

```

Arguments

<code>fit</code>	An estimated object of the class <code>mGASFit</code> . By default <code>fit = NULL</code> .
<code>T.sim</code>	numeric Length of the simulated time series.
<code>N</code>	numeric Cross sectional dimension (Only $N < 5$ supported for now).
<code>kappa</code>	numeric vector of unconditional level for the reparametrized vector of parameters.
<code>A</code>	matrix of coefficients of dimension $K \times K$ that premultiply the conditional score in the GAS updating recursion, see Details.

B	matrix of autoregressive coefficients of dimension $K \times K$, see Details.
Dist	character Label of the conditional distribution, see DistInfo .
ScalingType	character indicating the scaling mechanism for the conditional score. Only <code>ScalingType = "Identity"</code> is supported for multivariate distributions, see the function DistInfo .

Details

The function permits to simulate from an estimated `mGASFit` object. If `fit` is not provided, the user can specify a GAS model via the additional arguments `kappa`, `A`, `B`, `Dist` and `ScalingType`.

All the information regarding the supported multivariate conditional distributions can be investigated using the [DistInfo](#) function. The model is specified as:

$$y_t \sim p(y|\theta_t)$$

where θ_t is the vector of parameters for the density $p(y|\cdot)$. Note that, θ_t includes also those parameters that are not time-varying. The GAS recursion for θ_t is:

$$\theta_t = \Lambda(\tilde{\theta}_t)$$

$$\tilde{\theta}_t = \kappa + A * s_{t-1} + B * \tilde{\theta}_{t-1}$$

where $h(\cdot)$ is the mapping function (see [MultiMapParameters](#)) and $\tilde{\theta}_t$ is the vector of reparametrised parameters. The process is initialized at $\theta_1 = (I - B)^{-1}\kappa$, where κ is the Kappa vector. The vector s_t is the scaled score of $p(y|\cdot)$ with respect to θ_t . See [Ardia et. al. \(2016a\)](#) for further details.

Value

An object of the class `mGASSim`

Author(s)

Leopoldo Catania

References

Ardia D, Boudt K and Catania L (2016a). "Generalized Autoregressive Score Models in R: The GAS Package." <http://ssrn.com/abstract=2825380>.

Creal D, Koopman SJ, Lucas A (2013). "Generalized Autoregressive Score Models with Applications." *Journal of Applied Econometrics*, 28(5), 777-795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey AC (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press.

Examples

```

# Simulate from a GAS process with Multivariate Student-t conditional
# distribution, time-varying locations, scales, correlations
# and fixed shape parameter.
library("GAS")

set.seed(786)

T.sim = 1000 # Number of observations to simulate.
N      = 3   # Trivariate series.
Dist  = "mvt" # Conditional Multivariate Student-t distribution.

# Build unconditional vector of reparametrised parameters.

Mu = c(0.1, 0.2, 0.3) # Vector of location parameters (this is not transformed).
Phi = c(1.0, 1.2, 0.3) # Vector of scale parameters for the first, second and third variables.

Rho = c(0.1, 0.2, 0.3) # This represents vec(R), where R is the correlation matrix.
# Note that it is up to the user to ensure that vec(R) implies a
# proper correlation matrix.

Theta = c(Mu, Phi, Rho, 7) # Vector of parameters such that the degrees of freedom are 7.

kappa = MultiUnmapParameters(Theta, Dist, N)

A = matrix(0, length(kappa), length(kappa))

# Update scales and correlations, do not update locations and shape parameters.
diag(A) = c(0, 0, 0, 0.05, 0.01, 0.09, 0.01, 0.04, 0.07, 0)

B = matrix(0, length(kappa), length(kappa))

# Update scales and correlations, do not update locations and shape parameters.
diag(B) = c(0, 0, 0, 0.7, 0.7, 0.5, 0.94, 0.97, 0.92, 0)

Sim = MultiGASSim(fit = NULL, T.sim, N, kappa, A, B, Dist, ScalingType = "Identity")

Sim

```

MultiGASSpec

Multivariate GAS specification

Description

Specify the conditional distribution, scaling mechanism and time-varying parameters for multivariate GAS models.

Usage

```
MultiGASSpec(Dist = "mvnorm", ScalingType = "Identity",
             GASPar = list(location = FALSE, scale = TRUE,
                           correlation = FALSE, shape = FALSE),
             ScalarParameters = TRUE)
```

Arguments

Dist character indicating the label of the conditional distribution. Available distribution can be displayed using the function [DistInfo](#). Default value `Dist = "mvnorm"`.

ScalingType character indicating the scaling mechanism for the conditional score. Only `ScalingType = "Identity"` is supported for multivariate distributions.

GASPar list containing information about which parameters of the conditional distribution have to be time-varying. `location = TRUE` refers to the location parameters, `scale = TRUE` refers to the scale parameters, `shape = TRUE` refers to the shape parameter (e.g., the degree of freedom of the multivariate Student-t distribution), `correlation = TRUE` refers to the correlations. If the distribution specified in the `Dist` argument does not have, say, a shape parameter, the condition `shape = TRUE` is ignored.

ScalarParameters logical indicating if the parameters of the locations, scales and correlations dynamic have to be scalars or a diagonal matrices. By default `ScalarParameters = TRUE`.

Details

All the information regarding the supported multivariate conditional distributions can be investigated using the [DistInfo](#) function.

Value

An object of the class [mGASSpec](#)

Author(s)

Leopoldo Catania

References

Creal D, Koopman SJ, Lucas A (2011). "A Dynamic Multivariate Heavy-Tailed Model for Time-Varying Volatilities and Correlations." *Journal of Business & Economic Statistics*, 29(4), 552-563. doi: [10.1198/jbes.2011.10070](#).

Creal D, Koopman SJ, Lucas A (2013). "Generalized Autoregressive Score Models with Applications." *Journal of Applied Econometrics*, 28(5), 777-795. doi: [10.1002/jae.1279](#).

Harvey AC (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press.

Examples

```
# Specify a GAS model with multivariate Student-t
# conditional distribution and time-varying locations,
# scales and correlations parameters but constant shape parameter.

library("GAS")

GASSpec = MultiGASSpec(Dist = "mvt", ScalingType = "Identity",
                       GASPar = list(location = TRUE, scale = TRUE,
                                     correlation = TRUE, shape = FALSE))

GASSpec
```

MultiMapParameters *Mapping function for univariate distributions*

Description

Map unrestricted vector of parameters into the proper space. This function transforms the parameters updated using the GAS recursion into their proper space.

Usage

```
MultiMapParameters(Theta_tilde, Dist, N)
```

Arguments

Theta_tilde	numeric Vector of reparametrised parameters, see Details .
Dist	character Label of the conditional distribution, see DistInfo .
N	numeric Cross sectional dimension. Note that only $iN < 5$ is supported.

Details

The order of the parameters is generally: locations, scales, correlations, shape. When the distribution defined by `Dist` does not have, say, the shape parameter, this should be simply omitted. See also [DistInfo](#) for specific distributions.

Value

A numeric vector of parameters.

Author(s)

Leopoldo Catania

Examples

```

# Map unrestricted parameters for the Multivariate Student-t distribution with N=3
library("GAS")

N = 3

Dist = "mvt"

# Vector of location parameters (this is not transformed).
Mu_tilde = c(0.1,0.2,0.3)

# Vector of unrestricted scales parameters such that
# the scales will be equal to 1.0, 1.2 and 0.3, for the first, second and
# third variables, respectively.
Phi_tilde = c(log(1.0), log(1.2), log(0.3))

# The vector c(0.1,0.2,0.3) represents vec(R),
# where R is the correlation matrix.
# Note that is up to the user to ensure that
# vec(R) implies a proper correlation matrix
# The function UnMapR_C transforms vec(R) in a vector of unrestricted parameters. It is
# the inverse of the hyperspherical coordinates transformation.

Rho_tilde = UnMapR_C(c(0.1,0.2,0.3), N)

# Vector of unconditional reparametrised parameters such that the
# degrees of freedom are 7.
#
# LowerNu() prints the lower bound numerical parameter for the degree
# of freedom, see help(LowerNu)
#

Theta_tilde = c(Mu_tilde, Phi_tilde , Rho_tilde, log(7 - LowerNu()))

Theta = MultiMapParameters(Theta_tilde, Dist, N)

Theta

```

MultiUnmapParameters *Inverse of [MultiMapParameters](#)*

Description

Transform distribution parameters into the unrestricted parameters. The unrestricted vector of parameters is updated using the GAS recursion.

Usage

```
MultiUnmapParameters(Theta, Dist, N)
```

Arguments

Theta	numeric Vector parameters, see Details.
Dist	character Label of the conditional distribution, see DistInfo .
N	numeric Cross sectional dimension. Note that only $iN < 5$ is supported.

Details

The order of the parameters is generally: locations, scales, correlations, shape. When the distribution defined by `Dist` does not have, say, the shape parameter, this should be simply omitted. See also [DistInfo](#) for specific distributions.

Value

A numeric vector of parameters.

Author(s)

Leopoldo Catania

Examples

```
# Unmap parameters for the Multivariate Student-t distribution with N=3
library(GAS)

N = 3

Dist = "mvt"

# Vector of location parameters (this is not transformed).
Mu = c(0.1, 0.2, 0.3)

# Vector of scales parameters for the first, second and third variables.
Phi = c(1.0, 1.2, 0.3)

# This represents vec(R), where R is the correlation matrix.
# Note that it is up to the user to ensure that vec(R) implies a proper correlation matrix
Rho = c(0.1, 0.2, 0.3)

# Vector of parameters such that the degrees of freedom are 7.
Theta = c(Mu, Phi, Rho, 7)

Theta_tilde = MultiUnmapParameters(Theta, Dist, N)

Theta_tilde

# It works
all(abs(MultiMapParameters(Theta_tilde, Dist, N) - Theta) < 1e-16)
```

NumericalBounds	<i>Numerical bounds imposed in parameters transformation.</i>
-----------------	---

Description

Prints the numerical bounds.

Usage

UpperNu()
LowerNu()
UpperA()
LowerA()
UpperB()
LowerB()

Details

UpperNu() and LowerNu() print the numerical upper and lower bounds for the degree of freedom parameter of the Student-t distribution, std. (including also sstd and mvt).

UpperA() and LowerA() print the numerical upper and lower bounds for the score parameter in the GAS recursion. These bounds are applied to each diagonal element of the matrix A that premultiplies the scaled score.

UpperB() and LowerB() print the numerical upper and lower bounds for the autoregressive parameter in the GAS recursion. These bounds are applied to each diagonal element of the matrix B that premultiplies the past value of the parameters.

Value

Prints the numerical bounds.

Author(s)

Leopoldo Catania

Examples

UpperNu()
LowerNu()
UpperA()
LowerA()
UpperB()
LowerB()

PIT_test

*Goodness of fit for conditional densities***Description**

This function implements density goodness of fit procedure of Diebold et al. (1998).

Usage

```
PIT_test(U, G = 20, alpha = 0.05, plot = FALSE)
```

Arguments

U	numeric Vector of Probability Integral Transformation (PIT).
G	numeric Number of bins of the empirical cumulative density function of the PIT.
alpha	numeric Test confidence level.
plot	logical Indicates whether the histogram of the PIT has to be displayed. By default plot = FALSE.

Details

This function implements density goodness of fit procedure of Diebold et al. (1998). The test relies on the result that, if the series of estimated conditional distributions is the true one, then the PIT series evaluated accordingly are iid Unif(0, 1) distributed. The test of the iid Uniform(0, 1) assumption consists of two parts. The first part concerns the independent assumption, and it tests if all the conditional moments of the data, up to the fourth one, have been accounted for by the model, while the second part checks if the conditional distribution assumption is reliable by testing if the PITs are Uniform over the interval (0, 1). See also Jondeau and Rockinger (2006) and Vlaar and Palm (1993).

Value

A list with elements: (i) Hist and (ii) IID. The first element Hist concerns the test of the unconditional assumption of uniformity of the PIT, it is a list with elements:

- test Statistic test.
- crit The critical value of the test.
- pvalue The pvalue of the test.
- hist The histogram, evaluated using the [hist](#) function.
- confidence Approximated asymptotic confidence level.

The second element IID concerns the iid assumption, it is a list with elements:

- test A named numeric vector with elements: test1, test2, test3, test4 representing the Lagrange Multiplier test for the first four conditional moments of the PITs.

- `crit` The critical value of the test.
- `pvalue` A named numeric vector with elements: `pvalue1`, `pvalue2`, `pvalue3`, `pvalue4` representing the pvalues of the Lagrange Multiplier test for the first four conditional moments of the PITs.

Author(s)

Leopoldo Catania

References

Diebold FX, Gunther TA and Tay AS (1998). "Evaluating Density Forecasts with Applications to Financial Risk Management." *International Economic Review*, 39(4), 863-883. doi: [10.2307/2527342](https://doi.org/10.2307/2527342).

Jondeau E and Rockinger M (2006). "The Copula-Garch Model of Conditional Dependencies: An International Stock Market Application." *Journal of International Money and Finance*, 25(5), 827-853. doi: [10.1016/j.jimonfin.2006.04.007](https://doi.org/10.1016/j.jimonfin.2006.04.007).

Vlaar PJ and Palm FC (1993). "The Message in Weekly Exchange Rates in the European Monetary System: Mean Reversion, Conditional Heteroscedasticity, and Jumps." *Journal of Business & Economic Statistics*, 11(3), 351-360. doi: [10.1080/07350015.1993.10509963](https://doi.org/10.1080/07350015.1993.10509963).

Examples

```
data("StockIndices")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = FALSE, scale = TRUE,
                                   shape = FALSE))

FTSEMIB = StockIndices[, "FTSEMIB"]

Fit = UniGASFit(GASSpec, FTSEMIB)

U = pit(Fit)

Test = PIT_test(U, G = 20, alpha = 0.05, plot = TRUE)
```

plot-methods

Plot output from an object of the from the [GAS](#) package.

Description

This method provides plot functionalities for the [uGASFit](#), [mGASFit](#), [uGASSim](#), [mGASSim](#), [uGASFor](#), [mGASFor](#), [uGASRoll](#) and [mGASRoll](#) objects defined in the [GAS](#) package.

Usage

```
plot(x, y, ...)
```

```
PlotMenu(x)
```

Arguments

<code>x, y</code>	objects of class <code>uGASFit</code> , <code>mGASFit</code> , <code>uGASSim</code> , <code>mGASSim</code> , <code>uGASFor</code> , <code>mGASFor</code> , <code>uGASRoll</code> , <code>mGASRoll</code> .
<code>...</code>	additional arguments, see Details

Details

`plot` accepts the additional argument `numeric` argument which. By default `which = NULL`. If `which` is provided, `plot()` does not show the interactive menu and plot the corresponding option. The available options for each object class is printed by the function `PlotMenu(x)`. By default `which = NULL`, that is, `plot()` display an interactive menu.

Value

Displays a plot of an object of class `uGASFit`, `mGASFit`, `uGASSim`, `mGASSim`, `uGASFor`, `mGASFor`, `uGASRoll`, `mGASRoll`.

Author(s)

Leopoldo Catania

Examples

```
## Not run:
## Plot filtered estimates of a GAS model estimated on the
## Quarterly logarithmic change in percentage points of the Consumer Price Index data set (cpichg)
library("GAS")

data("cpichg")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = TRUE, scale = TRUE,
                                   shape = FALSE))

Fit = UniGASFit(GASSpec, cpichg)

plot(Fit, which = 1)

## End(Not run)
```

sp500ret	<i>Data: Daily logarithmic returns in percentage points of the S&P500 index from 1950-01-04 to 2016-06-24</i>
----------	---

Description

Daily logarithmic returns in percentage points of the S&P500 index from 1950-01-04 to 2016-06-24 obtained from yahoo finance.

Usage

```
data("sp500ret")
```

Format

A `xts` object of dimension 16,727 x 1 containing the daily logarithmic returns in percentage points from 1950-01-04 to 2016-06-24.

Source

Yahoo Finance

sp500rv	<i>Data: SP500 Daily 5 minutes Realized Volatility from 2000-01-03 to 2000-01-10</i>
---------	--

Description

Oxford-Man Institute Daily 5 minutes Realized Volatility from 2000-01-03 to 2000-01-10 for the SP500 Index available at <http://realized.oxford-man.ox.ac.uk/data>.

Usage

```
data("sp500rv")
```

Format

A `xts` object containing 4,310 observations from 2000-01-03 to 2000-01-10.

References

<http://realized.oxford-man.ox.ac.uk/data>

StockIndices	<i>Data: Daily logarithmic returns in percentage points of the DAX, FT-SEMIB and CAC40 from 2007-01-03 to 2016-06-24</i>
--------------	--

Description

Daily logarithmic returns in percentage points of the DAX, FTSEMIB and CAC40 from 2007-01-03 to 2016-06-24 obtained from Yahoo.

Usage

```
data("StockIndices")
```

Format

A matrix object of dimension 2,445 x 3 containing the daily logarithmic returns in percentage points from 2007-01-03 to 2016-06-24. Missing values are simply removed.

References

Yahoo finance.

tqdata	<i>Data from Bien et al (2011).</i>
--------	-------------------------------------

Description

From the readme.bnp.txt file in the JAE Data Archive available at <http://qed.econ.queensu.ca/jae/datasets/bien001/>:

The high-frequency data used in the paper come from the Trades and Quotation (TAQ) database. The data contains time-stamped quotations of Citicorp stock traded at the NYSE over the period from 20th February to 23rd February 2001.

In the study, 30-second bid and ask quote changes are constructed from the irregularly-spaced quote data. The study covers observations recorded from 9:35 EST until 16:00 EST.

The data contains 3080 rows and eight columns - in order:

1. year
2. month
3. day
4. time in number of seconds after the 9:35 EST
5. best ask quote
6. best bid quote
7. 30-second change of the ask quote in number of ticks
8. 30-second change of the bid quote in number of ticks.

Usage

```
data("tqdata")
```

Format

A `data.frame` object containing 3,080 observations.

References

Bien K, Nolte, I, Pohlmeier W (2011). "An Inflated Multivariate Integer Count Hurdle Model: An Application to Bid and Ask Quote Dynamics". *Journal of Applied Econometrics*, 26(4), 669-707. doi: [10.1002/jae.1122](https://doi.org/10.1002/jae.1122)

uGASFit

Class for the univariate GAS fitted object

Description

Class for the univariate GAS fitted object.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

ModelInfo: Object of class `list`. Contains information about the GAS specification:

- **Spec:** object of the class `uGASSpec` containing the GAS specification.
- **iT:** numeric number of observation.
- **elapsedTime:** numeric elapsed Time in seconds.

GASDyn: Object of class `list`. Contains: the series of filtered dynamic (`GASDyn$mTheta`) for the time-varying parameters, the series of scaled scores (`GASDyn$mInnovation`), the series of unrestricted filtered parameters (`GASDyn$mTheta_tilde`), the series of log densities (`GASDyn$vLLK`), the log likelihood evaluated at its optimum value (`GASDyn$dLLK`).

Estimates: Object of class `list`. Contains: `lParList` list of estimated parameters, optimiser object delivered from the optimization function, `StaticFit` ML estimates for the constant model, Inference inferential results for the estimated parameters.

Data: The user's data.

Testing: Statistical tests results.

Methods

- `show signature(object = 'uGASFit')`: print object information.
- `summary signature(object = 'uGASFit')`: Show summary.
- `plot signature(x = 'uGASFit', y = 'missing')`: Plot filtered dynamic and other estimated quantities.
- `getFilteredParameters signature(object = 'uGASFit')`: Extract filtered parameters.
- `getObs signature(object = 'uGASFit')`: Extract original observations.

- `coef signature(object = 'uGASFit')`: Returns a named vector of estimated coefficients. Also accepts the additional logical argument `do.list`. If `do.list = TRUE`, estimated coefficients are organized in a list with arguments: `vKappa` the intercept vector, `mA` the A system matrix, `mB` the B system matrix. By default, `do.list = FALSE`.
- `pit signature(object = 'uGASFit')`: Extract Probability Integral Transformation.
- `getMoments signature(object = 'uGASFit')`: Extract conditional moments.
- `residuals signature(object = 'uGASFit')`: Extract the residuals. Also accepts the additional logical argument `standardize`. If `standardize = TRUE`, residuals are standardized by the filtered standard deviation. By default `standardize = FALSE`.
- `convergence signature(object = 'uGASFit')`: Extract convergence information.
- `quantile signature(object = 'uGASSim')`: Compute quantiles of the filtered estimated density at each point in time. It accepts the additional argument `probs` representing the vector of probabilities.
- `ES signature(object = 'uGASSim')`: Compute Expected Shortfall of the filtered estimated density at each point in time. It accepts the additional argument `probs` representing the vector of probabilities.

Author(s)

Leopoldo Catania

uGASFor

Class for the univariate GAS forecast object

Description

Class for the univariate GAS forecast object.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

Forecast: Object of class `list`. Contains forecasts:

- `PointForecast`: matrix with parameters forecasts.
- `Moments`: matrix with centered moments forecasts.
- `vLS`: numeric Log Score (Predictive Log Likelihood).
- `vU`: numeric Out-of-sample Probability Integral Transformation (PIT).

Bands: array with confidence bands parameters forecasts. Available only if `Roll = TRUE`.

Draws: If `ReturnsDraws = TRUE` it is a `iH x iB` matrix of draws from the predictive distribution.

Info: list with forecast information.

Data: list with original data.

Methods

- `show signature(object = 'uGASFor')`: Show summary.
- `plot signature(x = 'uGASFor', y = 'missing')`: Plot forecasted quantities.
- `getForecast signature(object = 'uGASFor')`: Extract parameters forecast.
- `getObs signature(object = 'uGASFor')`: Extract original observations.
- `pit signature(object = 'uGASFor')`: Extract Probability Integral Transformation, only if `Roll = TRUE`.
- `quantile signature(object = 'uGASFor')`: Extract quantile forecasts. For multistep ahead prediction ES is computed by simulation and `ReturnsDraws = TRUE` should have been selected. It accepts the additional argument `probs` representing the vector of probabilities.
- `ES signature(object = 'uGASFor')`: Extract Expected Shortfall forecasts. For multistep ahead prediction ES is computed by simulation and `ReturnsDraws = TRUE` should have been selected. It accepts the additional argument `probs` representing the vector of probabilities.
- `getMoments signature(object = 'uGASFor')`: Extract moments forecasts.
- `LogScore signature(object = 'uGASFor')`: Extract Log Scores.

Author(s)

Leopoldo Catania

uGASRoll

Class for the univariate GAS rolling object

Description

Class for the univariate GAS rolling object.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

Forecast: Object of class `list`. Contains forecasts:

- `PointForecast`: matrix with parameters forecasts.
- `Moments`: matrix with centered moments forecasts.
- `vLS`: numeric Log Score (Predictive Log Likelihood).
- `vU`: numeric Out-of-sample Probability Integral Transformation (PIT).

Info: list with forecast information.

Data: list with original data.

Testing: Statistical tests results.

Methods

- `show signature(object = 'uGASRoll')`: Show summary.
- `plot signature(x = 'uGASRoll', y = 'missing')`: Plot forecasted quantities.
- `getForecast signature(object = 'uGASRoll')`: Extract parameters forecast.
- `getObs signature(object = 'uGASRoll')`: Extract original observations.
- `pit signature(object = 'uGASRoll')`: Extract Probability Integral Transformation, only if `Roll = TRUE`
- `quantile signature(object = 'uGASRoll')`: Extract quantile forecasts. It accepts the additional argument `probs` representing the vector of probabilities.
- `ES signature(object = 'uGASRoll')`: Extract Expected Shortfall forecasts. It accepts the additional argument `probs` representing the vector of probabilities.
- `getMoments signature(object = 'uGASRoll')`: Extract moments forecasts.
- `LogScore signature(object = 'uGASRoll')`: Extract Log Scores.
- `residuals signature(object = 'uGASRoll')`: Extract the forecast errors. Also accepts the additional logical argument `standardize`. If `standardize = TRUE`, forecast errors are standardized by the forecast standard deviation. By default `standardize = FALSE`.
- `coef signature(object = 'uGASFit')`: Returns a matrix of estimated coefficients. Each row of the matrix corresponds to a refit of the model during the forecast period according to the `RefitEvery` argument provided in the [UniGASRoll](#) function. Also accepts the additional logical argument `do.list`. If `do.list = TRUE`, estimated coefficients are organized in a list of lists according to the `RefitEvery` argument provided in the [UniGASRoll](#) function. Each list is populated by three arguments: `vKappa` the intercept vector, `mA` the A system matrix, `mB` the B system matrix. By default, `do.list = FALSE`.

Author(s)

Leopoldo Catania

uGASSim

Class for Univariate GAS Simulation

Description

Class for Univariate GAS model Simulation.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

ModelInfo: Object of class `list`. Contains information about the univariate GAS specification:

- `iT` numeric Time length of simulated observations.
- `iK` numeric Number of (possibly) time-varying parameters implied by the distributional assumption.
- `vKappa` numeric Vector of unconditional level for the reparametrised vector of parameters.
- `mA` matrix Of coefficients of dimension `iK` x `iK` that premultiply the conditional score in the GAS updating recursion.
- `mB` matrix Of autoregressive coefficients of dimension `iK` x `iK`.
- `Dist` character Label of the conditional distribution, see [DistInfo](#)
- `ScalingType` character Representing the scaling mechanism for the conditional score, see [DistInfo](#).

GASDyn: Object of class `list`. Contains: the series of simulated parameters (`GASDyn$mTheta`), the series of scaled scores (`GASDyn$mInnovation`), the series of unrestricted simulated parameters (`GASDyn$mTheta_tilde`), the series of log densities (`GASDyn$vLLK`), the log likelihood evaluated at its optimum value (`GASDyn$dLLK`).

Data: Object of class `numeric`. Vector of length `iT` of simulated data.

Methods

- `show signature(object = 'uGASSim')`: Show summary.
- `plot signature(x = 'uGASSim', y = 'missing')`: Plot simulated data and parameters.
- `getFilteredParameters signature(object = 'uGASSim')`: Extract simulated parameters.
- `getObs signature(object = 'uGASSim')`: Extract simulated observations.
- `coef signature(object = 'uGASSim')`: Extract delivered coefficients.
- `quantile signature(object = 'uGASSim')`: Compute quantiles of the filtered simulated density at each point in time. It accepts the additional argument `probs` representing the vector of probabilities.
- `ES signature(object = 'uGASSim')`: Compute the Expected Shortfall of the filtered simulated density at each point in time. It accepts the additional argument `probs` representing the vector of probabilities.

Author(s)

Leopoldo Catania

uGASSpec

Class for the univariate GAS model specification

Description

Class for the univariate GAS model specification.

Objects from the Class

A virtual Class: No objects may be created from it.

Slots

Spec: Object of class list. Contains information about the univariate GAS specification:

- Dist: character containing the conditional distribution assumption.
- ScalingType: character indicating the scaling mechanism for the conditional score.
- iK: numeric representing the number of (possibly) time-varying parameters implied by the distributional assumption.
- GASPar list with elements: location, scale, skewness, shape, shape2.

Methods

- show signature(object = 'uGASSpec'): Show summary.

Author(s)

Leopoldo Catania

UniGASFit

Estimate univariate GAS models

Description

Estimate univariate GAS models by Maximum Likelihood.

Usage

```
UniGASFit(GASSpec, data, fn.optimizer = fn.optim, Compute.SE = TRUE)
```

Arguments

GASSpec	An object of the class <code>uGASSpec</code> created using the function <code>UniGASSpec</code> .
data	numeric vector of length $T \times 1$ containing the time series of observations. It can also be an object of the class <code>ts</code> , <code>xts</code> or <code>zoo</code> .
fn.optimizer	function. This is a generic optimization function that can be provided by the user. By default <code>fn.optimizer = fn.optim</code> where <code>fn.optim</code> is a wrapper to the <code>optim</code> function. See Details for user defined optimization routines.
Compute.SE	logical. Should asymptotic Standard Errors be computed? By default <code>Compute.SE = TRUE</code>

Details

Maximum Likelihood estimation of GAS models is an on-going research topic. General results are reported by Blasques et al. (2014b), Blasques et al. (2014a) and Harvey (2013), while results for specific models have been derived by Blasques et al. (2014c) and Andres (2014).

Starting values for the optimizer are chosen in the following way: (i) estimate the static version of the model (i.e., with $A = 0$ and $B = 0$) and set the initial value of the intercept parameter accordingly, and (ii) perform a grid search for the coefficients contained in A and B . Further technical details are presented in Section 3.2 of Ardia et. al. (2016a).

The user is free to employ his/her own optimization routine via the `fn.optimizer` argument. `fn.optimizer` accepts a function object. The user provided optimizer has to satisfy strict requirements. The arguments of the `fn.optimizer` are : i) `par0` a vector of starting values, ii) `data` the data provided, iii) `GASSpec` an object of the class `uGASSpec`, and iv) `FUN` the likelihood function. The output of `fn.optimizer` has to be an object of the class `list` with four named elements: i) `pars`: a numeric vector where the estimated parameters are stored, ii) `value`: a numeric containing the value of the negative log likelihood evaluated at its minimum, iii) `hessian`, a numeric matrix containing the Hessian matrix evaluated at the minimum of the negative log likelihood, this is used for inferential purposes, and iv) `convergence` a numeric variable reporting information about the convergence of the optimization. This quantity is printed by the `show()` and `summary()` methods. `convergence = 0` has to indicate successful completion.

The user is allowed to not include the last two elements of the output of the `fn.optimizer` function, that is, the values `hessian = NULL` and `convergence = NULL` are admissible. In the case of `hessian = NULL`, the Hessian matrix is evaluated numerically using the `hessian` function in the `numDeriv` package of Gilbert and Varadhan (2016). If the provided hessian is not positive definite, a try with the hessian evaluation used by the BFGS quasi-Newton implementation in the function `optim` is made.

By default, the `optim` optimizer with `method = "BFGS"` is employed.

Value

An object of the class `uGASFit`

Author(s)

Leopoldo Catania

References

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- Harvey AC (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press.
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Examples

```
## Not run:
# Specify an univariate GAS model with Student-t
# conditional distribution and time-varying scale.
library("GAS")

data("sp500ret")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = FALSE, scale = TRUE,
                                   shape = FALSE))

Fit = UniGASFit(GASSpec, sp500ret)

Fit
```

```

# Estimate the model with a different optimizer.
# Assume we want to use the Nelder and Mead optimization provided by
# the optim() function, we create
# the wrapper fn.NM.optim in this way

fn.NM.optim <- function(par0, data, GASSpec, FUN) {

  optimizer = optim(par0, FUN, data = data, GASSpec = GASSpec, method = "Nelder-Mead",
                    control = list(trace = 0), hessian = TRUE)

  out = list(pars = optimizer$par,
            value = optimizer$value,
            hessian = optimizer$hessian,
            convergence = optimizer$convergence)

  return(out)
}

Fit.NM.optim = UniGASFit(GASSpec, sp500ret, fn.optimizer = fn.NM.optim )

Fit.NM.optim

# Estimate time-varying Negative Binomial distribution for the Goals dataset.
# Let's use the gosolnp() optimizer for the time-varying model estimation and
# the solnp() optimizer for estimation of the static model for the choice of
# the starting values. The logical is(GASSpec, "list") is TRUE when the function
# is evaluated for the choice of starting values, and FALSE when the function
# is evaluated for the time-varying model.
# We can also make use of parallel computation calling a cluster object defined
# in the Global environment.

library("Rsolnp")
fn.gosolnp <- function(par0, data, GASSpec, FUN) {

  if (is(GASSpec, "list")) {

    optimiser = suppressWarnings(solnp(par0, FUN, data = data,
                                      GASSpec = GASSpec,
                                      control = list(trace = 0)))

  } else {

    cluster = get("cluster", envir = globalenv())

    optimiser = suppressWarnings(gosolnp(
      pars = NULL,
      fun = FUN, data = data, cluster = cluster,
      GASSpec = GASSpec,
      n.sim = 100000,
      n.restarts = 10,
      LB = c(-5, -2, -2, -2),

```

```

        UB = c(5, 8, 3.0, 5.0))
    )
}

out = list(pars = optimiser$pars,
          value = tail(optimiser$values, 1),
          hessian = optimiser$hessian,
          convergence = optimiser$convergence)

return(out)
}

data("Goals")

library("parallel")

cluster = makeCluster(2)

GASSpec = UniGASSpec(Dist = "negbin", ScalingType = "Inv",
                    GASPar = list(location = TRUE, scale = FALSE))

vY = na.omit(Goals[, 1])

Fit = UniGASFit(GASSpec, vY, fn.optimizer = fn.gosolnp)

Fit

stopCluster(cluster)
rm("cluster")

## End(Not run)

```

 UniGASFor

Forecast with univariate GAS models

Description

Forecast with univariate GAS models. The one-step ahead prediction of the conditional density is available in closed form. The multi-step ahead prediction is performed by simulation as detailed in Blasques et al. (2016).

Usage

```

UniGASFor(uGASFit, H = NULL, Roll = FALSE, out = NULL, B = 10000,
          Bands = c(0.1, 0.15, 0.85, 0.9), ReturnDraws = FALSE)

```


Arguments

uGASFit	An object of the class <code>uGASFit</code> created using the function <code>UniGASFit</code> .
H	numeric Forecast horizon. Ignored if <code>Roll = TRUE</code> .
Roll	logical Forecast should be made using a rolling procedure ? Note that, if <code>Roll = TRUE</code> , then <code>out</code> has to be specified.
out	numeric Vector of out-of-sample observation for rolling forecast.
B	numeric Number of draws from the H-step ahead distribution if <code>Roll = FALSE</code> .
Bands	numeric Vector of probabilities representing the confidence band levels for multi-step ahead parameters forecasts. Only if <code>Roll = FALSE</code> .
ReturnDraws	logical Return the draws from the multi-step ahead predictive distribution when <code>Roll = FALSE</code> ?

Value

An object of the class `uGASFor`.

Author(s)

Leopoldo Catania

References

Blasques F, Koopman SJ, Lasak K, and Lucas, A (2016). "In-sample Confidence Bands and Out-of-Sample Forecast Bands for Time-Varying Parameters in Observation-Driven Models." *International Journal of Forecasting*, 32(3), 875-887. doi: [10.1016/j.ijforecast.2016.04.002](https://doi.org/10.1016/j.ijforecast.2016.04.002).

Examples

```
# Specify an univariate GAS model with Student-t
# conditional distribution and time-varying location, scale and shape parameter

# Inflation Forecast

set.seed(123)

data("cpichg")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = TRUE, scale = TRUE, shape = FALSE))

# Perform H-step ahead forecast with confidence bands

Fit = UniGASFit(GASSpec, cpichg)
Forecast = UniGASFor(Fit, H = 12)

Forecast

# Perform 1-Step ahead rolling forecast
```

```

InsampleData = cpichg[1:250]
OutSampleData = cpichg[251:276]

Fit = UniGASFit(GASSpec, InsampleData)

Forecast = UniGASFor(Fit, Roll = TRUE, out = OutSampleData)

Forecast

```

UniGASRoll

Rolling forecast with univariate GAS models

Description

One-step ahead rolling forecasts with model re-estimation. The function also reports several quantity for backtesting for point and density forecasts.

Usage

```

UniGASRoll(data, GASSpec, ForecastLength = 500, Nstart = NULL,
            RefitEvery = 23, RefitWindow = c("moving", "recursive"),
            cluster = NULL, Compute.SE = FALSE, ...)

```

Arguments

<code>data</code>	numeric vector containing the time series of observations.
<code>GASSpec</code>	An object of the class <code>uGASSpec</code> created using the function <code>UniGASSpec</code> .
<code>ForecastLength</code>	numeric Length of the out-of-sample.
<code>Nstart</code>	numeric Period when perform the first forecast. Ignored if <code>ForecastLength</code> is supplied.
<code>RefitEvery</code>	numeric Number of periods before model coefficients re-estimation.
<code>RefitWindow</code>	character Type of window. If <code>RefitWindow = "recursive"</code> all the observations are used when the model is re-estimated. If <code>RefitWindow = "moving"</code> old observations are eliminated.
<code>cluster</code>	A cluster object created calling using the <code>paralell</code> package. If supplied parallel processing is used to speed up the computations.
<code>Compute.SE</code>	logical. Should asymptotic Standard Errors be computed? By default <code>Compute.SE = FALSE</code>
<code>...</code>	Additional arguments for <code>UniGASFit</code>

Value

An object of the class `uGASRoll`.

Author(s)

Leopoldo Catania

Examples

```

# Specify an univariate GAS model with Student-t
# conditional distribution and time-varying location, scale and shape parameter

# Inflation Forecast

data("cpichg")
help(cpichg)

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = TRUE, scale = TRUE, shape = FALSE))

# Perform 1-step ahead rolling forecast with refit
library("parallel")

Roll = UniGASRoll(cpichg, GASSpec, ForecastLength = 50,
                 RefitEvery = 12, RefitWindow = c("moving"))

Roll

```

UniGASSim

Simulate Univariate GAS processes

Description

Simulate Univariate GAS processes.

Usage

```

UniGASSim(fit = NULL, T.sim = 1000,
          kappa = NULL, A = NULL, B = NULL, Dist = NULL, ScalingType = NULL)

```

Arguments

<code>fit</code>	An estimated object of the class <code>uGASFit</code> . By default <code>fit = NULL</code> .
<code>T.sim</code>	numeric Length of the simulated time series.
<code>kappa</code>	numeric Vector of unconditional level for the reparametrised vector of parameters. Only used if <code>fit = NULL</code>
<code>A</code>	matrix Of coefficients of dimension $K \times K$ that premultiply the conditional score in the GAS updating recursion, see Details. Only used if <code>fit = NULL</code>
<code>B</code>	matrix Of autoregressive coefficients of dimension $K \times K$, see Details. Only used if <code>fit = NULL</code>
<code>Dist</code>	character Label of the conditional distribution, see DistInfo . Only used if <code>fit = NULL</code>

ScalingType character Indicating the scaling mechanism for the conditional score. Possible choices are "Identity", "Inv", "InvSqrt". Note that, for some distribution only `ScalingType = "Identity"` is supported, see the function `DistInfo`. When `ScalingType = "InvSqrt"` the inverse of the Cholesky decomposition of the information matrix is used. Default value `ScalingType = "Identity"`. Only used if `fit = NULL`

Details

The function permits to simulate from an estimated `uGASFit` object. If `fit` is not provided, the user can specify a GAS model via the additional arguments `kappa`, `A`, `B`, `Dist` and `ScalingType`.

All the information regarding the supported univariate conditional distributions can be investigated using the `DistInfo` function. The model is specified as

$$y_t \sim p(y|\theta_t)$$

, where θ_t is the vector of parameters for the density $p(y|\cdot)$. Note that, θ_t includes also those parameters that are not time-varying. The GAS recursion for θ_t is

$$\theta_t = \Lambda(\tilde{\theta}_t)$$

,

$$\tilde{\theta}_t = \kappa + A * s_{t-1} + B * \tilde{\theta}_{t-1}$$

, where $\Lambda(\cdot)$ is the mapping function (see `UniMapParameters`) and $\tilde{\theta}_t$ is the vector of reparametrised parameters. The process is initialized at $\theta_1 = (I - B)^{-1}\kappa$, where κ is the `vKappa` vector. The vector s_t is the scaled score of $p(y|\cdot)$ with respect to θ_t . See Ardia et. al. (2016a) for further details.

Value

An object of the class `uGASSim`.

Author(s)

Leopoldo Catania

References

Ardia D, Boudt K and Catania L (2016a). "Generalized Autoregressive Score Models in R: The GAS Package." <http://ssrn.com/abstract=2825380>.

Creal D, Koopman SJ, Lucas A (2013). "Generalized Autoregressive Score Models with Applications." *Journal of Applied Econometrics*, 28(5), 777-795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey AC (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press.

Examples

```

# Simulate from a GAS process with Student-t conditional
# distribution, time-varying location, scale and fixed shape parameter.

library(GAS)

set.seed(786)

T.sim = 1000 # number of observations to simulate
Dist = "std" # conditional Student-t distribution

# vector of unconditional reparametrised parameters such that, the unconditional level of
#  $\theta_t$  is (0, 1.5, 7), i.e. location = 0, scale = 1.5,
# degrees of freedom = 7.

kappa = c(0.0, log(1.5), log(7-2.01))

# in this way we specify that the shape parameter is constant while the score
# coefficients for the location and the scale
# parameters are 0.001 and 0.01, respectively.

A = matrix(c(0.001, 0.0, 0.0,
            0.0, 0.01, 0.0,
            0.0, 0.0, 0.0), 3, byrow = TRUE)

B = matrix(c(0.7, 0.0, 0.0,
            0.0, 0.98, 0.0,
            0.0, 0.0, 0.0), 3, byrow = TRUE) # Matrix of autoregressive parameters.

Sim = UniGASSim(fit = NULL, T.sim, kappa, A, B, Dist, ScalingType = "Identity")

Sim

```

UniGASSpec

Univariate GAS specification

Description

Specify the conditional distribution, scaling mechanism and time-varying parameters for univariate GAS models.

Usage

```

UniGASSpec(Dist = "norm", ScalingType = "Identity",
           GASPar = list(location = FALSE, scale = TRUE,
                        skewness = FALSE, shape = FALSE, shape2 = FALSE))

```

Arguments

Dist	character Indicating the label of the conditional distribution. Available distribution can be displayed using the function DistInfo . Default value Dist = "norm".
ScalingType	character Indicating the scaling mechanism for the conditional score. Possible choices are "Identity", "Inv", "InvSqrt". Note that, for some distribution only ScalingType = "Identity" is supported, see the function DistInfo . When ScalingType = "InvSqrt" the inverse of the cholesky decomposition of the information matrix is used. Default value ScalingType = "Identity".
GASPar	list Containing information about which parameters of the conditional distribution have to be time-varying. location = TRUE refers to the location parameter, scale = TRUE refers to the scale parameter, skewness = TRUE refers to the parameter controlling the skewness, shape = TRUE refers to the shape parameter (e.g. the degree of freedom of the Student-t distribution), shape2 = TRUE refers to the second shape parameter. If the distribution specified in the Dist argument does not have, say, a shape parameter, the condition shape = TRUE or shape = FALSE is ignored. Note that, for some distributions, these labels are not strictly related to their literal statistical meaning. Indeed, for the Exponential distribution exp, the term location indicates the usual intensity rate parameter. See the DistInfo function for more details.

Details

All the information regarding the supported univariate conditional distributions can be investigated using the [DistInfo](#) function.

Value

An object of the class [uGASSpec](#).

Author(s)

Leopoldo Catania

References

Ardia D, Boudt K and Catania L (2016). "Generalized Autoregressive Score Models in R: The GAS Package." <http://ssrn.com/abstract=2825380>.

Creal D, Koopman SJ, Lucas A (2013). "Generalized Autoregressive Score Models with Applications." *Journal of Applied Econometrics*, 28(5), 777-795. doi: [10.1002/jae.1279](https://doi.org/10.1002/jae.1279).

Harvey AC (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge University Press.

Examples

```
# Specify an univariate GAS model with Student-t
# conditional distribution and time-varying location, scale and shape parameter
library("GAS")

GASSpec = UniGASSpec(Dist = "std", ScalingType = "Identity",
                    GASPar = list(location = TRUE,
                                   scale = TRUE, shape = TRUE))

GASSpec
```

UniMapParameters	<i>Mapping function for univariate distributions</i>
------------------	--

Description

Map unrestricted vector of parameters into the proper space. This function transforms the parameters updated using the GAS recursion into their proper space.

Usage

```
UniMapParameters(Theta_tilde, Dist)
```

Arguments

Theta_tilde	numeric Vector of reparametrised parameters, see Details .
Dist	character Label of the conditional distribution, see DistInfo .

Details

The order of the parameters is generally: location, scale, skewness, shape, shape2. When the distribution defined by `Dist` does not have, say, the shape parameter, this should be simply omitted. See also [DistInfo](#) for specific distributions.

Value

A numeric vector of parameters.

Author(s)

Leopoldo Catania

Examples

```

# Map unrestricted parameters for the Student-t distribution.
library("GAS")

Dist = "std"

# Vector of unconditional reparametrised parameters such that,
# Theta = c(0, 1.5 ,7), i.e., location = 0, scale = 1.5,
# degrees of freedom = 7.

# LowerNu() prints the lower bound numerical parameter for the degree
# of freedom, see help(LowerNu).

Theta_tilde = c(0.1, log(1.5), log(7 - LowerNu()))

Theta = UniMapParameters(Theta_tilde, Dist)

Theta

```

UniUnmapParameters *Unmapping function for univariate distributions, i.e. inverse of*
UniMapParameters

Description

Transform distribution parameters into the unrestricted parameters. The unrestricted vector of parameters is updated using the GAS recursion.

Usage

```
UniUnmapParameters(Theta, Dist)
```

Arguments

Theta numeric Vector of parameters, see Details.
Dist character Label of the conditional distribution, see [DistInfo](#).

Details

The order of the parameters is generally: location, scale, skewness, shape, shape2. When the distribution defined by Dist does not have, say, the shape parameter, this should be simply omitted. See also [DistInfo](#) for specific distributions.

Value

A numeric vector of parameters.

Author(s)

Leopoldo Catania

Examples

```
# Unmap parameters for the Student-t distribution
library("GAS")

Dist = "std"

# Vector of parameters such that,
# Theta = c(0, 1.5, 7), i.e., location = 0, scale = 1.5,
# degrees of freedom = 7.

Theta = c(0.1, 1.5, 7)

Theta_tilde = UniUnmapParameters(Theta, Dist)

Theta_tilde

# It works.
all(abs(UniMapParameters(Theta_tilde, Dist) - Theta) < 1e-16)
```

usunp	<i>US Monthly Civilian Unemployment Rate (UNRATE) from 1948-01-01 to 2016-05-01</i>
-------	---

Description

From <https://fred.stlouisfed.org/series/UNRATE>: The unemployment rate represents the number of unemployed as a percentage of the labor force. Labor force data are restricted to people 16 years of age and older, who currently reside in 1 of the 50 states or the District of Columbia, who do not reside in institutions (e.g., penal and mental facilities, homes for the aged), and who are not on active duty in the Armed Forces.

Usage

```
data("usunp")
```

Format

A [xts](#) object containing 821 observations from 1948-01-01 to 2016-05-01.

References

US. Bureau of Labor Statistics, Civilian Unemployment Rate [UNRATE], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/UNRATE>, July 2, 2016.

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