Package ‘ald’

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Description It provides the density, distribution function, quantile function, random number generator, likelihood function, moments and Maximum Likelihood estimators for a given sample, all this for the three parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999). This is a special case of the skewed family of distributions available in Galarza et.al. (2017) <doi:10.1002/sta4.140> useful for quantile regression.

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`ald-package` *The Asymmetric Laplace Distribution*

Description

It provides the density, distribution function, quantile function, random number generator, likelihood function, moments and Maximum Likelihood estimators for a given sample, all this for the three parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression.

Details

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References

Koenker, R., Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. J. Amer. Statist. Assoc. 94(3):1296-1309.

Yu, K. & Moyeed, R. (2001). Bayesian quantile regression. Statistics & Probability Letters, 54(4), 437-447.

Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. Communications in Statistics-Theory and Methods, 34(9-10), 1867-1879.

See Also

ALD,momentsALD,likALD,mleALD

Examples

```
## Let's plot an Asymmetric Laplace Distribution!

##Density
sseq = seq(-40,80,0.5)
dens = dALD(y=sseq,mu=50,sigma=3,p=0.75)
plot(sseq,dens,type="l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")
```

```
## Distribution Function
df = pALD(q=sseq,mu=50,sigma=3,p=0.75)
plot(sseq,df,type="l",lwd=2,col="blue",xlab="x",ylab="F(x)", main="ALD Distribution function")
abline(h=1,lty=2)

##Inverse Distribution Function
prob = seq(0,1,length.out = 1000)
idf = qALD(prob=prob,mu=50,sigma=3,p=0.75)
plot(prob,idf,type="l",lwd=2,col="gray30",xlab="x",ylab=expression(F^{-1}~(x)))
title(main="ALD Inverse Distribution function")
abline(v=c(0,1),lty=2)

#Random Sample Histogram
sample = rALD(n=10000,mu=50,sigma=3,p=0.75)
hist(sample,breaks = 70,freq = FALSE,ylim=c(0,max(dens)),main="")
title(main="Histogram and True density")
lines(sseq,dens,col="red",lwd=2)

## Let's compute the MLE's

param = c(-323,40,0.9)
y = rALD(10000,mu = param[1],sigma = param[2],p = param[3])  #A random sample
res = mleALD(y)

#Comparing
cbind(param,res$par)

#Let's plot

seqq = seq(min(y),max(y),length.out = 1000)
dens = dALD(y=seqq,mu=res$par[1],sigma=res$par[2],p=res$par[3])
hist(y,breaks=50,freq = FALSE,ylim=c(0,max(dens)))
lines(seqq,dens,type="l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")
```

`ALD` *The Asymmetric Laplace Distribution*

Description

Density, distribution function, quantile function and random generation for a Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to `mu`, scale parameter `sigma` and skewness parameter `p` This is a special case of the skewed family of distributions in Galarza (2016) available in `lqr::SKD`.

Usage

```
dALD(y, mu = 0, sigma = 1, p = 0.5)
pALD(q, mu = 0, sigma = 1, p = 0.5, lower.tail = TRUE)
```

```
qALD(prob, mu = 0, sigma = 1, p = 0.5, lower.tail = TRUE)
rALD(n, mu = 0, sigma = 1, p = 0.5)
```

Arguments

`y,q`	vector of quantiles.
`prob`	vector of probabilities.
`n`	number of observations.
`mu`	location parameter.
`sigma`	scale parameter.
`p`	skewness parameter.
`lower.tail`	logical; if TRUE (default), probabilities are P[X $\leq$ x] otherwise, P[X > x].

Details

If `mu`, `sigma` or `p` are not specified they assume the default values of 0, 1 and 0.5, respectively, belonging to the Symmetric Standard Laplace Distribution denoted by $ALD(0, 1, 0.5)$.

As discussed in Koenker and Machado (1999) and Yu and Moyeed (2001) we say that a random variable Y is distributed as an ALD with location parameter μ, scale parameter $\sigma > 0$ and skewness parameter p in (0,1), if its probability density function (pdf) is given by

$$f(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp -\rho_p(\frac{y-\mu}{\sigma})$$

where $\rho_p(.)$ is the so called check (or loss) function defined by

$$\rho_p(u) = u(p - I_{u<0})$$

, with $I_.$ denoting the usual indicator function. This distribution is denoted by $ALD(\mu, \sigma, p)$ and it's p-th quantile is equal to μ.

The scale parameter `sigma` must be positive and non zero. The skew parameter `p` must be between zero and one (0<p<1).

Value

`dALD` gives the density, `pALD` gives the distribution function, `qALD` gives the quantile function, and `rALD` generates a random sample.

The length of the result is determined by `n` for `rALD`, and is the maximum of the lengths of the numerical arguments for the other functions `dALD`, `pALD` and `qALD`.

Note

The numerical arguments other than `n` are recycled to the length of the result.

Author(s)

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References

Galarza Morales, C., Lachos Davila, V., Barbosa Cabral, C., and Castro Cepero, L. (2017) Robust quantile regression using a generalized class of skewed distributions. Stat,6: 113-130 doi: 10.1002/sta4.140.

Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. Communications in Statistics-Theory and Methods, 34(9-10), 1867-1879.

See Also

momentsALD,likALD,mleALD

Examples

```
## Let's plot an Asymmetric Laplace Distribution!

##Density
library(ald)
sseq = seq(-40,80,0.5)
dens = dALD(y=sseq,mu=50,sigma=3,p=0.75)
plot(sseq,dens,type = "l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")

#Look that is a special case of the skewed family in Galarza (2017)
# available in lqr package, dSKD(...,sigma = 2*3,dist = "laplace")

## Distribution Function
df = pALD(q=sseq,mu=50,sigma=3,p=0.75)
plot(sseq,df,type="l",lwd=2,col="blue",xlab="x",ylab="F(x)", main="ALD Distribution function")
abline(h=1,lty=2)

##Inverse Distribution Function
prob = seq(0,1,length.out = 1000)
idf = qALD(prob=prob,mu=50,sigma=3,p=0.75)
plot(prob,idf,type="l",lwd=2,col="gray30",xlab="x",ylab=expression(F^{-1}~(x)))
title(main="ALD Inverse Distribution function")
abline(v=c(0,1),lty=2)

#Random Sample Histogram
sample = rALD(n=10000,mu=50,sigma=3,p=0.75)
hist(sample,breaks = 70,freq = FALSE,ylim=c(0,max(dens)),main="")
title(main="Histogram and True density")
lines(sseq,dens,col="red",lwd=2)
```

---

likALD *Log-Likelihood function for the Asymmetric Laplace Distribution*

Description

Log-Likelihood function for the Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to `mu`, scale parameter `sigma` and skewness parameter `p`.

Usage

```
likALD(y, mu = 0, sigma = 1, p = 0.5, loglik = TRUE)
```

Arguments

y	observation vector.
mu	location parameter μ.
sigma	scale parameter σ.
p	skewness parameter p.
loglik	logical; if TRUE (default), the Log-likelihood is return, if not just the Likelihood.

Details

If mu, sigma or p are not specified they assume the default values of 0, 1 and 0.5, respectively, belonging to the Symmetric Standard Laplace Distribution denoted by $ALD(0, 1, 0.5)$.

As discussed in Koenker and Machado (1999) and Yu and Moyeed (2001) we say that a random variable Y is distributed as an ALD with location parameter μ, scale parameter $\sigma > 0$ and skewness parameter p in (0,1), if its probability density function (pdf) is given by

$$f(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp -\rho_p(\frac{y-\mu}{\sigma})$$

where $\rho_p(.)$ is the so called check (or loss) function defined by

$$\rho_p(u) = u(p - I_{u<0})$$

, with $I_.$ denoting the usual indicator function. Then the Log-likelihood function is given by

$$\sum_{i=1}^{n} log(\frac{p(1-p)}{\sigma} \exp -\rho_p(\frac{y_i-\mu}{\sigma}))$$

.

The scale parameter sigma must be positive and non zero. The skew parameter p must be between zero and one (0<p<1).

Value

likeALD returns the Log-likelihood by default and just the Likelihood if loglik = FALSE.

Author(s)

Christian E. Galarza <<cgalarza88@gmail.com>> and Victor H. Lachos <<hlachos@ime.unicamp.br>>

References

Koenker, R., Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. J. Amer. Statist. Assoc. 94(3):1296-1309.

Yu, K. & Moyeed, R. (2001). Bayesian quantile regression. Statistics & Probability Letters, 54(4), 437-447.

Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. Communications in Statistics-Theory and Methods, 34(9-10), 1867-1879.

See Also

ALD,momentsALD,mleALD

Examples

```
## Let's compute the log-likelihood for a given sample

y = rALD(n=1000)
loglik = likALD(y)

#Changing the true parameters the loglik must decrease
loglik2 = likALD(y,mu=10,sigma=2,p=0.3)

loglik;loglik2
if(loglik>loglik2){print("First parameters are Better")}
```

---

mleALD — *Maximum Likelihood Estimators (MLE) for the Asymmetric Laplace Distribution*

Description

Maximum Likelihood Estimators (MLE) for the Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to `mu`, scale parameter `sigma` and skewness parameter `p`.

Usage

```
mleALD(y, initial = NA)
```

Arguments

`y`	observation vector.
`initial`	optional vector of initial values c(μ, σ, p).

Details

The algorithm computes iteratevely the MLE's via the combination of the MLE expressions for μ and σ, and then maximizing with rescpect to p the Log-likelihood function (`likALD`) using the well known `optimize` R function. By default the tolerance is 10^-5 for all parameters.

Value

The function returns a list with two objects

- `iter` iterations to reach convergence.
- `par` vector of Maximum Likelihood Estimators.

Author(s)

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References

Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. Communications in Statistics-Theory and Methods, 34(9-10), 1867-1879.

See Also

ALD,momentsALD,likALD

Examples

```
## Let's try this function

param = c(-323,40,0.9)
y = rALD(10000,mu = param[1],sigma = param[2],p = param[3])  #A random sample
res = mleALD(y)

#Comparing
cbind(param,res$par)

#Let's plot

seqq = seq(min(y),max(y),length.out = 1000)
dens = dALD(y=seqq,mu=res$par[1],sigma=res$par[2],p=res$par[3])
hist(y,breaks=50,freq = FALSE,ylim=c(0,max(dens)))
lines(seqq,dens,type="l",lwd=2,col="red",xlab="x",ylab="f(x)", main="ALD Density function")
```

---

momentsALD *Moments for the Asymmetric Laplace Distribution*

Description

Mean, variance, skewness, kurtosis, central moments w.r.t `mu` and first absolute central moment for the Three-Parameter Asymmetric Laplace Distribution defined in Koenker and Machado (1999) useful for quantile regression with location parameter equal to `mu`, scale parameter `sigma` and skewness parameter `p`.

Usage

```
meanALD(mu=0,sigma=1,p=0.5)
varALD(mu=0,sigma=1,p=0.5)
skewALD(mu=0,sigma=1,p=0.5)
kurtALD(mu=0,sigma=1,p=0.5)
momentALD(k=1,mu=0,sigma=1,p=0.5)
absALD(sigma=1,p=0.5)
```

Arguments

k	moment number.
mu	location parameter μ.
sigma	scale parameter σ.
p	skewness parameter p.

Details

If mu, sigma or p are not specified they assume the default values of 0, 1 and 0.5, respectively, belonging to the Symmetric Standard Laplace Distribution denoted by $ALD(0, 1, 0.5)$.

As discussed in Koenker and Machado (1999) and Yu and Moyeed (2001) we say that a random variable Y is distributed as an ALD with location parameter μ, scale parameter $\sigma > 0$ and skewness parameter p in (0,1), if its probability density function (pdf) is given by

$$f(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp -\rho_p(\frac{y-\mu}{\sigma})$$

where $\rho_p(.)$ is the so called check (or loss) function defined by

$$\rho_p(u) = u(p - I_{u<0})$$

, with $I_.$ denoting the usual indicator function. This distribution is denoted by $ALD(\mu, \sigma, p)$ and it's pth quantile is equal to μ. The scale parameter sigma must be positive and non zero. The skew parameter p must be between zero and one (0<p<1).

Value

meanALD gives the mean, varALD gives the variance, skewALD gives the skewness, kurtALD gives the kurtosis, momentALD gives the kth central moment, i.e., $E(y-\mu)^k$ and absALD gives the first absolute central moment denoted by $E|y-\mu|$.

Author(s)

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References

Koenker, R., Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. J. Amer. Statist. Assoc. 94(3):1296-1309.

Yu, K. & Moyeed, R. (2001). Bayesian quantile regression. Statistics & Probability Letters, 54(4), 437-447.

Yu, K., & Zhang, J. (2005). A three-parameter asymmetric Laplace distribution and its extension. Communications in Statistics-Theory and Methods, 34(9-10), 1867-1879.

See Also

ALD,likALD,mleALD

Examples

```
## Let's compute some moments for a Symmetric Standard Laplace Distribution.

#Third raw moment
momentALD(k=3,mu=0,sigma=1,p=0.5)

#The well known mean, variance, skewness and kurtosis
meanALD(mu=0,sigma=1,p=0.5)
varALD(mu=0,sigma=1,p=0.5)
skewALD(mu=0,sigma=1,p=0.5)
kurtALD(mu=0,sigma=1,p=0.5)

# and this guy
absALD(sigma=1,p=0.5)
```

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