Using the doRNG package

doRNG package – Version 1.8.6

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Introduction

Research reproducibility is an issue of concern, e.g. in bioinformatics [3, 9, 4]. Some analyses require multiple independent runs to be performed, or are amenable to a splitand-reduce scheme. For example, some optimisation algorithms are run multiple times from different random starting points, and the result that achieves the least approximation error is selected. The *foreach* package¹ [7] provides a very convenient way to perform parallel computations, with different parallel environments such as MPI or Redis, using a transparent loop-like syntax:

¹https://cran.r-project.org/package=foreach

For each parallel environment a *backend* is implemented as a specialised **%dopar%** operator, which performs the setup and pre/post-processing specifically required by the environment (e.g. export of variable to each worker). The **foreach** function and the **%dopar%** operator handle the generic parameter dispatch when the task are split between worker processes, as well as the reduce step – when the results are returned to the master worker.

When stochastic computations are involved, special random number generators must be used to ensure that the separate computations are indeed statistically independent – unless otherwise wanted – and that the loop is reproducible. In particular, standard %dopar% loops are not reproducible:

```
# with standard %dopar%: foreach loops are not reproducible
set.seed(123)
res <- foreach(i=1:5) %dopar% { runif(3) }
set.seed(123)
res2 <- foreach(i=1:5) %dopar% { runif(3) }
identical(res, res2)
## [1] FALSE</pre>
```

A random number generator commonly used to achieve reproducibility is the combined multiple-recursive generator from L'Ecuyer [5]. This generator can generate independent random streams, from a 6-length numeric seed. The idea is then to generate a sequence of random stream of the same length as the number of iteration (i.e. tasks) and use a different stream when computing each one of them.

The doRNG package² [2] provides convenient ways to implement reproducible parallel **foreach** loops, independently of the parallel backend used to perform the computation. We illustrate its use, showing how non-reproducible loops can be made reproducible, even when tasks are not scheduled in the same way in two separate set of runs, e.g. when the

²https://cran.r-project.org/package=doRNG

workers do not get to compute the same number of tasks or the number of workers is different. The package has been tested with the $doParallel^3$ and $doMPI^4$ packages [10, 1], but should work with other backends such as provided by the doRedis package⁵ [6].

1 The %dorng% operator

The *doRNG* package defines a new generic operator, **%dorng%**, to be used with foreach loops, instead of the standard %dopar%. Loops that use this operator are *de facto* reproducible.

```
# load the doRNG package
library(doRNG)
## Loading required package: rngtools
# using %dorng%: loops _are_ reproducible
set.seed(123)
res <- foreach(i=1:5) %dorng% { runif(3) }
set.seed(123)
res2 <- foreach(i=1:5) %dorng% { runif(3) }
identical(res, res2)
```

[1] TRUE

1.1 How it works

For a loop with N iterations, the "dorng" operator internally performs the following tasks:

- 1. generate a sequence of random seeds $(S_i)_{1 \le i \le N}$ for the *R* random number generator "L'Ecuyer-CMRG" [5], using the function nextRNGStream from the *parallel* package⁶ [8], which ensure the different RNG streams are statistically independent;
- 2. modify the loop's R expression so that the random number generator is set to "L'Ecuyer-CMRG" at the beginning of each iteration, and is seeded with consecutive seeds in (S_n) : iteration i is seeded with S_i , $1 \le i \le N$;
- 3. call the standard **%dopar%** operator, which in turn calls the relevant (i.e. registered) foreach parallel backend;
- 4. store the whole sequence of random seeds as an attribute in the result object:

³https://cran.r-project.org/package=doParallel

⁴https://cran.r-project.org/package=doMPI

⁵https://cran.r-project.org/package=doRedis

⁶https://cran.r-project.org/package=parallel

```
attr(res, 'rng')
## [[1]]
## [1]
            10407
                    642048078
                                 81368183 -2093158836
                                                        506506973 1421492218 -1906381517
##
## [[2]]
            10407 1340772676 -1389246211 -999053355
                                                       -953732024
                                                                   1888105061
## [1]
                                                                               2010658538
##
## [[3]]
## [1]
            10407 -1318496690 -948316584
                                            683309249
                                                       -990823268 -1895972179 1275914972
##
## [[4]]
                    524763474 1715794407 1887051490 -1833874283
                                                                    494155061 -1221391662
            10407
## [1]
##
## [[5]]
## [1]
             10407 -1816009034 -580124020 1603250023 817712173 190009158 -706984535
```

1.2 Seeding computations

Sequences of random streams for "L'Ecuyer-CMRG" are generated using a 6-length integer seed, e.g.,:

nextRNGStream(c(407L, 1:6))

[1] 407 -447371532 542750874 -935969228 -269326340 701604884 -1748056907

However, the **%dorng%** operator provides alternative – convenient – ways of seeding reproducible loops.

- set.seed: as shown above, calling set.seed before the loop ensure reproducibility of the results, using a single integer as a seed. The actual 6-length seed is then generated with an internal call to RNGkind("L'Ecuyer-CMRG").
- .options.RNG with single integer: the %dorng% operator support options that can be passed in the foreach statement, containing arguments for the internal call to set.seed:

```
# use a single numeric as a seed
s <- foreach(i=1:5, .options.RNG=123) %dorng% { runif(3) }
s2 <- foreach(i=1:5, .options.RNG=123) %dorng% { runif(3) }
identical(s, s2)</pre>
```

```
## [1] TRUE
```

Note: calling set.seed before the loop is equivalent to passing the seed in .options.RNG. See Section 1.3 for more details.

The kind of Normal generator may also be passed in .options.RNG:

```
## Pass the Normal RNG kind to use within the loop
# results are identical if not using the Normal kind in the loop
optsN <- list(123, normal.kind="Ahrens")
resN.U <- foreach(i=1:5, .options.RNG=optsN) %dorng% { runif(3) }
identical(resN.U[1:5], res[1:5])
## [1] TRUE
# Results are different if the Normal kind is used and is not the same
resN <- foreach(i=1:5, .options.RNG=123) %dorng% { rnorm(3) }
resN1 <- foreach(i=1:5, .options.RNG=optsN) %dorng% { rnorm(3) }
resN2 <- foreach(i=1:5, .options.RNG=optsN) %dorng% { rnorm(3) }
identical(resN[1:5], resN1[1:5])
## [1] FALSE
identical(resN1[1:5], resN2[1:5])
## [1] TRUE
```

.options.RNG with 6-length: the actual 6-length integer seed used for the first RNG stream may be passed via options.RNG:

```
# use a 6-length numeric
s <- foreach(i=1:5, .options.RNG=1:6) %dorng% { runif(3) }</pre>
attr(s, 'rng')[1:3]
## [[1]]
## [1] 10407
                1
                       2
                             3
                                   4
                                          5
                                                6
##
## [[2]]
## [1]
             10407 -447371532
                                 542750874
                                            -935969228 -269326340
                                                                      701604884 -1748056907
##
## [[3]]
## [1]
             10407 311773008 -1393648596
                                             433058656 -545474683 2059732357
                                                                                   994549473
```

.options.RNG with 7-length: a 7-length integer seed may also be passed via options.RNG, which is useful to seed a loop with the value of .Random.seed as used in some iteration of another loop⁷:

```
# use a 7-length numeric, used as first value for .Random.seed
seed <- attr(res, 'rng')[[2]]
s <- foreach(i=1:5, .options.RNG=seed) %dorng% { runif(3) }
identical(s[1:4], res[2:5])
```

⁷Note that the RNG kind is then always required to be the "L'Ecuyer-CMRG", i.e. the first element of the seed must have unit 7 (e.g. 407 or 107).

[1] TRUE

.options.RNG with complete sequence of seeds: the complete description of the sequence of seeds to be used may be passed via options.RNG, as a list or a matrix with the seeds in columns. This is useful to seed a loop exactly as desired, e.g. using an RNG other than "L'Ecuyer-CMRG", or using different RNG kinds in each iteration, which probably have different seed length, in order to compare their stochastic properties. It also allows to reproduce %dorng% loops without knowing their seeding details:

```
# reproduce previous %dorng% loop
s <- foreach(i=1:5, .options.RNG=res) %dorng% { runif(3) }</pre>
identical(s, res)
## [1] TRUE
## use completely custom sequence of seeds (e.g. using RNG "Marsaglia-Multicarry")
# as a matrix
seedM <- rbind(rep(401, 5), mapply(rep, 1:5, 2))</pre>
seedM
##
        [,1] [,2] [,3] [,4] [,5]
## [1,] 401 401 401 401 401
## [2,]
        1 2
                   3 4
                               5
## [3,]
                2
                     3
                           4
           1
                                5
sM <- foreach(i=1:5, .options.RNG=seedM) %dorng% { runif(3) }</pre>
# same seeds passed as a list
seedL <- lapply(seq(ncol(seedM)), function(i) seedM[,i])</pre>
sL <- foreach(i=1:5, .options.RNG=seedL) %dorng% { runif(3) }</pre>
identical(sL, sM)
## [1] TRUE
```

1.3 Difference between set.seed and .options.RNG

While it is equivalent to seed %dorng% loops with set.seed and .options.RNG, it is important to note that the result depends on the current RNG kind ⁸:

```
# default RNG kind
RNGkind('default')
def <- foreach(i=1:5, .options.RNG=123) %dorng% { runif(3) }</pre>
```

⁸See Section 7 about a bug in versions ; 1.4 on this feature.

```
# Marsaglia-Multicarry
RNGkind('Marsaglia')
## Warning in RNGkind("Marsaglia"): RNGkind: Marsaglia-Multicarry has poor statistical
properties
mars <- foreach(i=1:5, .options.RNG=123) %dorng% { runif(3) }
identical(def, mars)
## [1] FALSE
# revert to default RNG kind
RNGkind('default')</pre>
```

This is a "normal" behaviour, which is a side-effect of the expected equivalence between set.seed and .options.RNG. This should not be a problem for reproducibility though, as R RNGs are stable across versions, and loops are most of the time used with the default RNG settings. In order to ensure seeding is independent from the current RNG, one has to pass a 7-length numeric seed to .options.RNG, which is then used directly as a value for .Random.seed (see below).

2 Parallel environment independence

An important feature of **%dorng%** loops is that their result is independent of the underlying parallel physical settings. Two separate runs seeded with the same value will always produce the same results. Whether they use the same number of worker processes, parallel backend or task scheduling does not influence the final result. This also applies to computations performed sequentially with the doSEQ backend. The following code illustrates this feature using 2 or 3 workers.

```
# define a stochastic task to perform
task <- function() c(pid=Sys.getpid(), val=runif(1))
# using the previously registered cluster with 2 workers
set.seed(123)
res_2workers <- foreach(i=1:5, .combine=rbind) %dorng% {
        task()
}
# stop cluster
stopCluster(cl)
# Sequential computation
registerDoSEQ()
set.seed(123)
res_seq <- foreach(i=1:5, .combine=rbind) %dorng% {
        task()</pre>
```

```
}
#
# Using 3 workers
# NB: if re-running this vignette you should edit to force using 3 here
cl <- makeCluster( if(isManualVignette()) 3 else 2)</pre>
length(cl)
## [1] 2
# register new cluster
registerDoParallel(cl)
set.seed(123)
res_3workers <- foreach(i=1:5, .combine=rbind) %dorng% {</pre>
       task()
}
# task schedule is different
pid <- rbind(res1=res_seq[,1], res_2workers[,1], res2=res_3workers[,1])</pre>
storage.mode(pid) <- 'integer'</pre>
pid
##
       result.1 result.2 result.3 result.4 result.5
## res1 2767062 2767062 2767062 2767062 2767062
##
       2767094 2767095 2767094 2767095 2767094
## res2 2767126 2767125 2767126 2767125 2767126
# results are identical
identical(res_seq[,2], res_2workers[,2]) && identical(res_2workers[,2], res_3workers[,2])
```

[1] TRUE

3 Reproducible %dopar% loops

The *doRNG* package also provides a non-invasive way to convert **%dopar%** loops into reproducible loops, i.e. without changing their actual definition. It is useful to quickly ensure the reproducibility of existing code or functions whose definition is not accessible (e.g. from other packages). This is achieved by registering the **doRNG** backend:

```
set.seed(123)
res <- foreach(i=1:5) %dorng% { runif(3) }
registerDoRNG(123)
res_dopar <- foreach(i=1:5) %dopar% { runif(3) }
identical(res_dopar, res)
## [1] TRUE
attr(res_dopar, 'rng')</pre>
```

[[1]] ## [1] 10407 642048078 81368183 -2093158836 506506973 1421492218 -1906381517 ## ## [[2]] ## [1] 10407 1340772676 -1389246211 -999053355 -953732024 1888105061 2010658538 ## ## [[3]] ## [1] 10407 -1318496690 -948316584 683309249 -990823268 -1895972179 1275914972 ## ## [[4]] ## [1] 10407 524763474 1715794407 1887051490 -1833874283 494155061 -1221391662 ## ## [[5]] ## [1] 10407 -1816009034 -580124020 1603250023 817712173 190009158 -706984535

4 Reproducibile sets of loops

Sequences of multiple loops are reproducible, whether using the **%dorng%** operator or the registered doRNG backend:

```
set.seed(456)
s1 <- foreach(i=1:5) %dorng% { runif(3) }</pre>
s2 <- foreach(i=1:5) %dorng% { runif(3) }</pre>
# the two loops do not use the same streams: different results
identical(s1, s2)
## [1] FALSE
# but the sequence of loops is reproducible as a whole
set.seed(456)
r1 <- foreach(i=1:5) %dorng% { runif(3) }</pre>
r2 <- foreach(i=1:5) %dorng% { runif(3) }</pre>
identical(r1, s1) && identical(r2, s2)
## [1] TRUE
# one can equivalently register the doRNG backend and use %dopar%
registerDoRNG(456)
r1 <- foreach(i=1:5) %dopar% { runif(3) }</pre>
r2 <- foreach(i=1:5) %dopar% { runif(3) }</pre>
identical(r1, s1) && identical(r2, s2)
## [1] TRUE
```

5 Nested and conditional loops

Nested and conditional foreach loops are currently not supported and generate an error:

```
# nested loop
try( foreach(i=1:10) %:% foreach(j=1:i) %dorng% { rnorm(1) } )
## Error: nested/conditional foreach loops are not supported yet.
## See the package's vignette for a work around.
# conditional loop
try( foreach(i=1:10) %:% when(i %% 2 == 0) %dorng% { rnorm(1) } )
## Error: nested/conditional foreach loops are not supported yet.
## See the package's vignette for a work around.
```

In this section, we propose a general work around for this kind of loops, that will eventually be incorporated in the **%dorng%** operator – when I find out how to mimic its behaviour from the operator itself.

5.1 Nested loops

The idea is to create a sequence of RNG seeds before the outer loop, and use each of them successively to set the RNG in the inner loop – which is exactly what **%dorng%** does for simple loops:

```
# doRNG must not be registered
registerDoParallel(cl)
# generate sequence of seeds of length the number of computations
n <- 10; p <- 5
rng <- RNGseq( n * p, 1234)</pre>
# run standard nested foreach loop
res <- foreach(i=1:n) %:% foreach(j=1:p, r=rng[(i-1)*p + 1:p]) %dopar% {</pre>
        # set RNG seed
    rngtools::setRNG(r)
    # do your own computation ...
    c(i, j, rnorm(1))
}
# Compare against the equivalent sequential computations
k <- 1
res2 <- foreach(i=1:n) %:% foreach(j=1:p) %do%{</pre>
   # set seed
       rngtools::setRNG(rng[[k]])
  k <- k + 1
```

```
# do your own computation ...
        c(i, j, rnorm(1))
}
stopifnot( identical(res, res2) )
```

The following is a more complex example with unequal - but **known** *a priori* - numbers of iterations performed in the inner loops:

```
# generate sequence of seeds of length the number of computations
n <- 10
rng <- RNGseq( n * (n+1) / 2, 1234)</pre>
# run standard nested foreach loop
res <- foreach(i=1:n) %:% foreach(j=1:i, r=rng[(i-1)*i/2 + 1:i]) %dopar%{
        # set RNG seed
        rngtools::setRNG(r)
        # do your own computation ...
        c(i, j, rnorm(1))
}
# Compare against the equivalent sequential computations
k <- 1
res2 <- foreach(i=1:n) %:% foreach(j=1:i) %do%{</pre>
        # set seed
        rngtools::setRNG(rng[[k]])
        k <- k + 1
        # do your own computation ...
        c(i, j, rnorm(1))
}
stopifnot( identical(res, res2) )
```

5.2 Conditional loops

The work around used for nested loops applies to conditional loops that use the when() clause. It ensures that the RNG seed use for a given inner iteration does not depend on the filter, but only on its index in the unconditional-unfolded loop:

```
# un-conditional single loop
resAll <- foreach(i=1:n, .options.RNG=1234) %dorng%{
    # do your own computation ...
    c(i, rnorm(1))
}</pre>
```

```
# generate sequence of RNG
rng <- RNGseq(n, 1234)
# conditional loop: even iterations
resEven <- foreach(i=1:n, r=rng) %:% when(i %% 2 == 0) %dopar%{
        # set RNG seed
        rngtools::setRNG(r)
        # do your own computation ...
        c(i, rnorm(1))
}
# conditional loop: odd iterations
resOdd <- foreach(i=1:n, r=rng) %:% when(i %% 2 == 1) %dopar%{
        # set RNG seed
        rngtools::setRNG(r)
        # do your own computation ...
        c(i, rnorm(1))
}
# conditional loop: only first 2 and last 2
resFL <- foreach(i=1:n, r=rng) %:% when(i %in% c(1,2,n-1,n)) %dopar%{
        # set RNG seed
        rngtools::setRNG(r)
        # do your own computation ...
        c(i, rnorm(1))
}
# compare results
stopifnot( identical(resAll[seq(2,n,by=2)], resEven) )
stopifnot( identical(resAll[seq(1,n,by=2)], resOdd) )
stopifnot( identical(resAll[c(1,2,n-1,n)], resFL) )
```

5.3 Nested conditional loops

Conditional nested loops may use the same work around, as shown in this intricate example:

```
# generate sequence of seeds of length the number of computations
n <- 10
rng <- RNGseq( n * (n+1) / 2, 1234)
# run standard nested foreach loop
res <- foreach(i=1:n) %:% when(i %% 2 == 0) %:% foreach(j=1:i, r=rng[(i-1)*i/2 + 1:i]) %dopar%{</pre>
```

```
# set RNG seed
rngtools::setRNG(r)
# do your own computation ...
c(i, j, rnorm(1))
}
# Compare against the equivalent sequential computations
k <- 1
resAll <- foreach(i=1:n) %:% foreach(j=1:i) %do%{
    # set seed
    rngtools::setRNG(rng[[k]])
    k <- k + 1
    # do your own computation ...
c(i, j, rnorm(1))
}
stopifnot( identical(resAll[seq(2,n,by=2)], res) )</pre>
```

6 Performance overhead

The extra setup performed by the <code>%dorng%</code> operator leads to a slight performance overhead, which might be significant for very quick computations, but should not be a problem for realistic computations. The benchmarks below show that a <code>%dorng%</code> loop may take up to two seconds more than the equivalent <code>%dopar%</code> loop, which is not significant in practice, where parallelised computations typically take several minutes.

```
# load rbenchmark
library(rbenchmark)
# comparison is done on sequential computations
registerDoSEQ()
rPar <- function(n, s=0){ foreach(i=1:n) %dopar% { Sys.sleep(s) } }
rRNG <- function(n, s=0){ foreach(i=1:n) %dorng% { Sys.sleep(s) } }
# run benchmark
cmp <- benchmark(rPar(10), rRNG(10)
, rPar(25), rRNG(25)
, rPar(50), rRNG(50)
, rPar(50, .01), rRNG(50, .01)
, rPar(10, .05), rRNG(10, .05)
, replications=5)
# order by increasing elapsed time
cmp[order(cmp$elapsed), ]
```

##

test replications elapsed relative user.self sys.self user.child sys.child

## 1	rPar(10)	5	0.030	1.000	0.030	0.000	0	0
## 3	rPar(25)	5	0.055	1.833	0.056	0.000	0	0
## 2	rRNG(10)	5	0.069	2.300	0.070	0.000	0	0
## 5	rPar(50)	5	0.099	3.300	0.099	0.000	0	0
## 4	rRNG(25)	5	0.110	3.667	0.110	0.000	0	0
## 6	rRNG(50)	5	0.175	5.833	0.174	0.000	0	0
## 9	rPar(10, 0.05)	5	2.548	84.933	0.043	0.000	0	0
## 10	rRNG(10, 0.05)	5	2.568	85.600	0.062	0.000	0	0
## 7	rPar(50, 0.01)	5	2.687	89.567	0.149	0.008	0	0
## 8	rRNG(50, 0.01)	5	2.731	91.033	0.207	0.000	0	0

7 Known issues

- Nested and/or conditional foreach loops using the operator %:% are not currently not supported (see Section 5 for a work around).
- An error is thrown in doRNG 1.2.6, when the package iterators was not loaded, when used with foreach i = 1.4.
- There was a bug in versions prior to 1.4, which caused set.seed and .options.RNG not to be equivalent when the current RNG was "L'Ecuyer-CMRG". This behaviour can still be reproduced by setting:

doRNGversion('1.3')

To revert to the latest default behaviour:

doRNGversion(NULL)

8 News and changes

\begin{kframe}

```
{\ttfamily\noindent\color{warningcolor}{\#\# Warning in file(con, "{}r"{}): file("{}"{}) only supports open = "{}w+"{} and contained to the support of the s
```

Cleanup

stopCluster(cl)

Session information

```
R version 4.2.1 (2022-06-23)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 22.10
Matrix products: default
      /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.10.1
BLAS:
LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.10.1
Random number generation:
      L'Ecuyer-CMRG
RNG:
Normal: Inversion
Sample: Rejection
locale:
 [1] LC_CTYPE=en_IL
                      LC_NUMERIC=C
                                           LC_TIME=en_IL
                                                              LC_COLLATE=C
 [5] LC_MONETARY=en_IL LC_MESSAGES=en_IL LC_PAPER=en_IL
                                                              LC_NAME=C
                      LC_TELEPHONE=C
 [9] LC_ADDRESS=C
                                           LC_MEASUREMENT=en_IL LC_IDENTIFICATION=C
attached base packages:
[1] parallel stats
                    graphics grDevices utils
                                                datasets methods
                                                                     base
other attached packages:
[1] rbenchmark_1.0.0 doRNG_1.8.6
                                     rngtools_1.5.2
                                                     doParallel_1.0.17
[5] iterators_1.0.14 foreach_1.5.2
                                     knitr_1.41
                                                     pkgmaker_0.32.7
[9] registry_0.5-1
loaded via a namespace (and not attached):
 [1] codetools_0.2-18 withr_2.5.0 digest_0.6.31 assertthat_0.2.1 xtable_1.8-4
 [6] lifecycle_1.0.3 magrittr_2.0.3 evaluate_0.19 highr_0.10 rlang_1.0.6
[11] stringi_1.7.8 cli_3.5.0 vctrs_0.5.1
                                                    tools_4.2.1
                                                                  stringr_1.5.0
[16] glue_1.6.2 xfun_0.36
                                  compiler_4.2.1
```

References

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