

Package ‘nlirms’

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Title Non-Life Insurance Rate-Making System

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Description Design of non-life insurance rate-making system with a frequency and a severity component based on the a posteriori criteria. The rate-making system is a general form of bonus-malus system introduced by Lemaire (1995), <doi:10.1007/978-94-011-0631-3> and Frangos and Vrontos (2001), <doi:10.2143/AST.31.1.991>.

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Description

nlirms package design the non-life insurance rate-making system based on the posteriori frequency and severity component. The non-life insurance rate-making system is a actuarial system for pricing of non-life insurance contract. This system determinates fair rates and fair premiums for each of policyholders based on the claims frequency and severity history of policyholders in last years.

Details

usually a rate-making system is designed from three way:

1-rate-making system based on the frequency component

2-rate-making system based on the severity component

3-rate-making system based on the both frequency and severity component

nlirms package returns rate-Making system based on each of the three methods. in current version of nlirms package, five model can be applied for frequency component and also five model can be applied for severity component.

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- Stasinopoulos, D. M., & Rigby, R. A. (2007). Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, 23(7), 1-46.

Examples

```
# rate-Making system based on the posteriori frequency component
rmspfc(time = 5, claim = 5, fmu = .2, fsigma = 2, family = "PGA")

# rate-Making system based on the posteriori severity component
rmspsc(time=5, claim=5, sumsev=100, smu = 50, ssigma = 2, family ="EGA")

# rate-Making system based on the posteriori frequency and severity component
rmspfsc(time=5 ,claim=5, fmu = .1, fsigma = 2, sumsev=100, smu = 50, ssigma
= 3,family = list("PGA","EGA"))
```

enc.PGA	<i>Expected number of claims based on the Poisson-Gamma (Negetive Binomial) model</i>
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Description

enc.PGA() function gives the expected number of claims for a policyholder in the next time (for example in next year) with regards to the number of claims history of this policyholder in past time, based on the Poisson-Gamma (Negetive Binomial) model.

Usage

```
PGA(k, mu, sigma)
dPGA(k=1, mu=.1, sigma=2)
enc.PGA(time = 2, claim = 1, mu = .1, sigma = 2)
```

Arguments

k	vector of (non-negative integer) quantiles.
mu	positive mean parameter of the Poisson-Gamma (Negetive Binomial) distribution that it wil be obtained from fitting Poisson-Gamma (Negetive Binomial) distribution to the claim frequency data.
sigma	positive scale parameter of the Poisson-Gamma (Negetive Binomial) distribution that it will be obtained from fitting Poisson-Gamma (Negetive Binomial) distribution to the claim frequency data.
time	time period to claims freuency rate-making
claim	total number of claims that a policyholder had in past years

Details

Consider that the number of claims k , ($k=0,1,\dots$), given the parameter y , is distributed according to $\text{Poisson}(y)$, where y is denoting the different underlyin risk of each policyholder to have an accident. if y following the Gamma distribution, $y \sim \text{GA}(\mu, \sigma)$, with Parameterization that $E(y)=\mu$, Then by apply the Bayes theorem the unconditional distribution of the number of claims k will be Poisson-Gamma (Negetive Binomial) distribution, $\text{PIGA}(\mu, \sigma)$, with probability density function as the following form:

$$f(y) = \frac{\text{gamma}(\text{sigma} + k)}{\text{gamma}(k + 1) * \text{gamma}(\text{sigma})} * [\mu / (\mu + \text{sigma})]^k * [\text{sigma} / (\mu + \text{sigma})]^{\text{sigma}}$$

let $\text{claim} = k_1 + \dots + k_t$, is total number of claims that a policyholder had in t years, where k_i is the number of claims that the policyholder had in the year i , ($i=1, \dots, t=\text{time}$). by apply the Bayes theorem, the posterior structure function of y i.e. $f(y|k_1, \dots, k_t)$, for a policyholder with claim history k_1, \dots, k_t , following the Gamma distribution, $\text{GA}(\text{time} + (\text{sigma}/\mu), \text{claim} + \text{sigma})$. the expected number of claims based on the PGA model is equal to the mean of this posteriori distribution.

Value

enc.PGA() function return the expected number of claims based on the Poisson-Gamma (Negative Binomial) model. dPGA() function return the probability density of Poisson-Gamma (Negative Binomial) model.

Note

in enc.PGA() function μ and sigma must be grether than 0.

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Examples

```
dPGA(k=1, mu=.1, sigma=2)
enc.PGA(time = 2, claim = 1, mu = .1, sigma = 2)
time=1:5
enc.PGA(time = time, claim = 1, mu = .1, sigma = 2)
```

enc.PGIG	<i>Expected number of claims based on the Poisson-Generalized Inverse Gaussian (Sichel) model.</i>
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Description

enc.PGIG() function gives the expected number of claims for a policyholder in the next time (for example in next year) with regards to the number of claims history of this policyholder in past time, based on the Poisson-Generalized Inverse Gaussian (Sichel) model.

Usage

```
PGIG(k, mu, sigma, nu)
dPGIG(k = 1, mu = .1, sigma = 2, nu=1)
enc.PGIG(time = 2, claim = 1, mu =.1, sigma = 2,nu=1)
```

Arguments

k	vector of (non-negative integer) quantiles.
mu	positive mean parameter of the Poisson-Generalized Inverse Gaussian (Sichel) distribution that it will be obtained from fitting Poisson-Generalized Inverse Gaussian (Sichel) distribution to the claim frequency data.
sigma	positive dispersion parameter of the Poisson-Generalized Inverse Gaussian (Sichel) distribution that it will be obtained from fitting Poisson-Generalized Inverse Gaussian (Sichel) distribution to the claim frequency data.
nu	third parameter of the Poisson-Generalized Inverse Gaussian (Sichel) distribution that it will be obtained from fitting Poisson-Generalized Inverse Gaussian (Sichel) distribution to the claim frequency data.
time	time period to claims frequency rate-making.
claim	total number of claims that a policyholder had in past years.

Details

Consider that the number of claims k , ($k=0,1,\dots$), given the parameter y , is distributed according to $Poisson(y)$, where y is denoting the different underlying risk of each policyholder to have an accident. if y following the Generalized Inverse Gaussian distribution, $y \sim GIG(\mu, \sigma, \nu)$, with Parameterization that $E(y)=\mu$, Then by apply the Bayes theorem the unconditional distribution of the number of claims k will be Poisson-Generalized Inverse Gaussian (Sichel) distribution, $PGIG \sim (\mu, \sigma, \nu)$, with probability density function as the following form:

$$f(y) = \left[\frac{(\mu/c)^\nu * \text{besselK}(\alpha, k+\nu)}{\Gamma(k+1) * (\alpha * \sigma)^{k+\nu} * \text{besselK}(1/\sigma, \nu)} \right]$$

where $c = \text{besselK}(1/\sigma, \nu+1) / \text{besselK}(1/\sigma, \nu)$

$$\alpha^2 = \left[\frac{1}{\sigma^2} \right] + \left[\frac{2 * \mu}{c * \sigma} \right]$$

let $\text{claim} = k_1 + \dots + k_t$, is total number of claims that a policyholder had in t years, where k_i is the number of claims that the policyholder had in the year i , ($i=1, \dots, t=\text{time}$). by apply the Bayes

theorem, the posterior structure function of y i.e. $f(y|k_1, \dots, k_t)$, for a policyholder with claim history k_1, \dots, k_t , following the Generalized Gaussian distribution, $GIG(2 \cdot \text{time} + [c/(\sigma \cdot \mu)], c/(\sigma \cdot \mu), \nu + \text{claim})$. the expected number of claims based on the PGA model is equal to the mean of this posteriori distribution.

Value

enc.PGIG() function return the expected number of claims based on the Poisson-Generalized Inverse Gaussian (Sichel) model. dPGIG() function return the probability density of Poisson-Generalized Inverse Gaussian (Sichel) distribution.

Note

in enc.PGIG() function μ and σ must be grether than 0 and $-\infty < \nu < \infty$.

enc.PGIG() function for $\nu = -.5$, return the expected number of claims based on the Poisson-Inverse Gaussian model and dPGIG() function return the probability density of Poisson-Inverse Gaussian distribution.

enc.PGIG() function for $\nu = 0$, return the expected number of claims based on the Poisson-Harmonic model and dPGIG() function return the probability density of Poisson-Harmonic distribution.

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Examples

```

time=1:5
claim=0:4
# Expected number of claims based on the Poisson-Generalized Inverse Gaussian model
enc.PGIG(time = time, claim = claim, mu = .1, sigma = 2, nu=1)

# Expected number of claims based on the Poisson-Inverse Gaussian model
enc.PGIG(time = time, claim = claim, mu = .1, sigma = 2, nu=-.5)

# Expected number of claims based on the Poisson-Harmonic model
enc.PGIG(time = time, claim = claim, mu = .1, sigma = 2, nu=0)

```

enc.PIGA	<i>Expected number of claims based on the Poisson-Inverse Gamma model</i>
----------	---

Description

enc.PIGA() function gives the expected number of claims for a policyholder in the next time (for example in next year) with regards to the number of claims history of this policyholder in past time, based on the Poisson-Inverse Gamma model.

Usage

```

PIGA(k, mu, sigma)
dPIGA(k = 1, mu = .1, sigma = 2)
enc.PIGA(time = 2, claim = 1, mu = .1, sigma = 2)

```

Arguments

k	vector of (non-negative integer) quantiles.
mu	positive mean parameter of the Poisson-Inverse Gamma distribution that it will be obtained from fitting Poisson-Inverse Gamma distribution to the claim frequency data.
sigma	positive scale parameter of the Poisson-Inverse Gamma distribution that it will be obtained from fitting Poisson-Inverse Gamma distribution to the claim frequency data.
time	time period to claims frequency rate-making.
claim	total number of claims that a policyholder had in past years.

Details

Consider that the number of claims k , ($k=0,1,\dots$), given the parameter y , is distributed according to $\text{Poisson}(y)$, where y is denoting the different underlying risk of each policyholder to have an accident. If y following the Inverse Gamma distribution, $y \sim \text{IGA}(\mu, \sigma)$, with Parameterization that $E(y)=\mu$, Then by apply the Bayes theorem the unconditional distribution of the number of

claims k will be Poisson-Inverse Gamma distribution, $PIGA \sim (\mu, \sigma)$, with probability density function as the following form:

$$f(y) = 2 * \alpha^{(k+\sigma)/2} * \text{besselK}((4 * \alpha)^{.5}, k - \sigma) / [\text{gamma}(k+1) * \text{gamma}(\sigma)]$$

where $\alpha = \mu * (\sigma - 1)$. let $\text{claim} = k_1 + \dots + k_t$, is total number of claims that a policyholder had in t years, where k_i is the number of claims that the policyholder had in the year i , ($i=1, \dots, t=\text{time}$). by apply the Bayes theorem, the posterior structure function of y i.e. $f(y|k_1, \dots, k_t)$, for a policyholder with claim history k_1, \dots, k_t , following the Generalized Inverse Gaussian distribution, $GIG(2 * \text{time}, 2 * \mu * (\sigma - 1), \text{claim} - \sigma - 1)$. the expected number of claims based on the PIGA model is equal to the mean of this posteriori distribution.

Value

enc.PIGA() function return the expected number of claims based on the Poisson-Inverse Gamma model. dPIGA() function return the probability density of Poisson-Inverse Gamma distribution.

Note

in enc.PIGA() function μ must be grether than 0 and σ must be grether than 1.

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Examples

```
dPIGA(k=1, mu=.1, sigma=2)
enc.PIGA(time = 2, claim = 1, mu = .1, sigma = 2)
time=1:5
enc.PIGA(time = time, claim = 1, mu = .1, sigma = 2)
```

esc.EGA

*Expected severity of claims based on the Exponential-Gamma model***Description**

esc.EGA() function gives the expected severity of next claim for a policyholder with regards to the claims severity and frequency history of this policyholder in past time, based on the Exponential-Gamma model.

Usage

```
EGA(x, mu, sigma)
dEGA(x= 100, claim=1, mu = 50, sigma = 2)
esc.EGA(sumsev=100, claim =1, mu =50 , sigma = 2)
```

Arguments

x	vector of quantiles
mu	positive mean parameter of the Exponential-Gamma distribution that it will be obtained from fitting Exponential-Gamma distribution to the claim severity data.
sigma	positive scale parameter of the Exponential-Gamma distribution that it will be obtained from fitting Exponential-Gamma distribution to the claim severity data.
sumsev	sum severity of all claims that a policyholder had in past years
claim	total number of claims that a policyholder had in past years

Details

Consider that x be the size of the claim of each insured and z is the mean claim size for each insured, where conditional distribution of the size given the parameter z , is distributed according to $\text{exponential}(z)$. if z following the Gamma distribution, $z \sim \text{GA}(\mu, \sigma)$, with Parameterization that $E(z) = \mu$, Then by apply the Bayes theorem the unconditional distribution of claim size x will be exponential-Gamma model, $\text{EGA} \sim (\mu, \sigma)$, with probability density function as the following form:

$$f(x) = 2 * [(\sigma * x / \mu)^{(\sigma + 1) / 2}] * \text{besselK}((4 * x * \sigma / \mu)^{.5}, \sigma - 1) / \text{gamma}(\sigma)$$

let $\text{claim} = k_1 + \dots + k_t$, is the total number of claims and $\text{sumsev} = x_1 + \dots + x_{\text{claim}}$ is the total amount of claim size where x_i is the amount of claim size in the claim i , ($i = 1, \dots, i = \text{claim}$). by apply the Bayes theorem, the posterior structure function of x given the claims size history of the policyholder i.e. $f(x | x_1, \dots, x_{\text{claim}})$, following the Generalized Inverse Gaussian distribution, $\text{GIG}(2 * \sigma / \mu, 2 * \text{sumsev}, \sigma - \text{claim})$. the expected claim severity based on the EGIG model is equal to the mean of this posteriori distribution.

Value

esc.EGA() function return the expected severity of next claim based on the Exponential-Gamma model. dEGA() function return the probability density of Exponential-Gamma distribution.

Note

esc.EGA() does not dependent to time. in esc.EGA() function mu and sigma must be grether than 0.

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Examples

```
esc.EGA(sumsev=100 ,claim=1 , mu=50, sigma=2)
claim=0:5
esc.EGA(sumsev=100 ,claim=claim , mu=50, sigma=2)
```

esc.EGIG	<i>Expected severity of claims based on the Exponential-Generalized Inverse Gaussian model</i>
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Description

esc.EGIG() function gives the expected severity of next claim for a policyholder with regards to the claims severity and frequency history of this policyholder in past time, based on the Exponential-Generalized Inverse Gaussian model.

Usage

```
EGIG(x, mu, sigma, nu)
dEGIG(x= 100, claim=1, mu = 50, sigma = 2, nu=2)
esc.EGIG(sumsev=100 ,claim=1 , mu=50, sigma=2, nu=2)
```

Arguments

x	vector of quantiles
mu	positive mean parameter of the Exponential-Generalized Inverse Gaussian distribution that it will be obtained from fitting Exponential-Generalized Inverse Gaussian distribution to the claim severity data.
sigma	positive dispersion parameter of the Exponential-Generalized Inverse Gaussian distribution that it will be obtained from fitting Exponential-Generalized Inverse Gaussian distribution to the claim severity data.
nu	third parameter of the Exponential-Generalized Inverse Gaussian distribution that it will be obtained from fitting Exponential-Generalized Inverse Gaussian distribution to the claim severity data.
sumsev	sum severity of all claims that a policyholder had in past years
claim	total number of claims that a policyholder had in past years

Details

Consider that x be the size of the claim of each insured and z is the mean claim size for each insured, where conditional distribution of the size given the parameter z , is distributed according to exponential(z). if z following the Generalized Inverse Gaussian distribution, $z \sim \text{GIG}(\mu, \sigma, \nu)$, with Parameterization that $E(z) = \mu$, Then by apply the Bayes theorem the unconditional distribution of claim size x will be Exponential-Generalized Inverse Gaussian model, $\text{EGIG} \sim (\mu, \sigma, \nu)$, with probability density function as the following form:

$$f(y) = [c * (\sigma * \alpha)^{(\nu+1)/2} * \text{besselK}(\alpha, \nu-1)] / [\mu * \text{besselK}(1/\sigma, \nu)]$$

where $c = \text{besselK}(1/\sigma, \nu+1) / \text{besselK}(1/\sigma, \nu)$

$$\alpha^2 = [1/(\sigma^2)] + [2 * x * c / (\mu * \sigma)]$$

let $\text{claim} = k_1 + \dots + k_t$, is the total number of claims and $\text{sumsev} = x_1 + \dots + x_{\text{claim}}$ is the total amount of claim size where x_i is the amount of claim size in the claim i , ($i=1, \dots, i=\text{claim}$). by apply the Bayes

theorem, the posterior structure function of x given the claims size history of the policyholder i.e. $f(x|x_1, \dots, x_{\text{claim}})$, following the Generalized Inverse Gaussian distribution, $GIG(c/(\mu*\sigma), [\mu/(c*\sigma)]+2*\text{sumsev}, \nu-\text{claim})$. the expected claim severity based on the EGIG model is equal to the mean of this posteriori distribution.

Value

esc.EGIG() function return the expected severity of next claim based on the Exponential-Generalized Inverse Gaussian model. dEIGA() function return the probability density of Exponential-Generalized Inverse Gaussian distribution.

Note

esc.EGIG() does not dependent to time. in esc.EGIG() function μ and σ must be grether than 0 and $-\infty < \nu < \infty$.

esc.EGIG() function for $\nu = -0.5$, return the expected severity of next claim based on the Exponential-Inverse Gaussian model and dEIGA() function return the probability density of Exponential-Inverse Gaussian distribution.

esc.EGIG() function for $\nu = 0$, return the expected severity of next claim based on the Exponential-Harmonic model and dEIGA() function return the probability density of Exponential-Harmonic distribution.

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Examples

```

claim=0:5
# Expected severity of claims based on the Exponential-Generalized Inverse Gaussian model
esc.EGIG(sumsev=100 ,claim=1 , mu=50, sigma=2, nu=1)

# Expected severity of claims based on the Exponential-Inverse Gaussian model
esc.EGIG(sumsev=100 ,claim=1 , mu=50, sigma=2, nu=-.5)

# Expected severity of claims based on the Exponential-Harmonic model
esc.EGIG(sumsev=100 ,claim=claim , mu=50, sigma=2, nu=0)

```

esc.EIGA	<i>Expected severity of claims based on the Exponential-Inverse Gamma (Pareto) model</i>
----------	--

Description

esc.EIGA() function gives the expected severity of next claim for a policyholder with regards to the claims severity and frequency history of this policyholder in past time, based on the Exponential-Inverse Gamma (Pareto) model.

Usage

```

EIGA(x, mu, sigma)
dEIGA(x= 100, claim=1, mu = 50, sigma = 2)
esc.EIGA(sumsev=100, claim =1, mu =50 , sigma = 2)

```

Arguments

x	vector of quantiles
mu	positive mean parameter of the Exponential-Inverse Gamma (Pareto) distribution that it will be obtained from fitting Exponential-Inverse Gamma (Pareto) distribution to the claim severity data.
sigma	positive scale parameter of the Exponential-Inverse Gamma (Pareto) distribution that it will be obtained from fitting Exponential-Inverse Gamma (Pareto) distribution to the claim severity data.
sumsev	sum severity of all claims that a policyholder had in past years
claim	total number of claims that a policyholder had in past years

Details

Consider that x be the size of the claim of each insured and z is the mean claim size for each insured, where conditional distribution of the size given the parameter z , is distributed according to $\text{exponential}(z)$. if z following the Inverse Gamma distribution, $z \sim \text{IGA}(\mu, \sigma)$, with Parameterization that $E(z) = \mu$, Then by apply the Bayes theorem the unconditional distribution of claim size x will be exponential-Inverse Gamma (Pareto) model, $\text{EIGA} \sim (\mu, \sigma)$, with probability density function as the following form:

$$f(x)=\sigma*[(\mu*(\sigma-1))^{\sigma}]/[(x+\mu*(\sigma-1))^{\sigma+1}]$$

let $\text{claim}=k_1+ \dots+k_t$, is the total number of claims and $\text{sumsev}=x_1+ \dots+x_{\text{claim}}$ is the total amount of claim size where x_i is the amount of claim size in the claim i , ($i=1, \dots, i=\text{claim}$). by apply the Bayes theorem, the posterior structure function of x given the claims size history of the policyholder i.e. $f(x|x_1, \dots, x_{\text{claim}})$, following the Inverse Gamma distribution, $\text{IGA}(\text{claim}+\sigma, \text{sumsev}*\mu*(\sigma-1))$. the expected claim severity based on the EIGA model is equal to the mean of this posteriori distribution.

Value

esc.EIGA() function return the expected severity of next claim based on the Exponential-Inverse Gamma (Pareto) model. dEIGA() function return the probability density of Exponential-Inverse Gamma (Pareto) distribution.

Note

esc.EIGA() does not dependent to time. in esc.EIGA() function μ must be grether than 0 and σ must be grether than 1.

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Examples

```
esc.EIGA(sumsev=100 ,claim=1 , mu=50, sigma=2)
claim=0:5
esc.EIGA(sumsev=100 ,claim=claim , mu=50, sigma=2)
```

nlirms.family

Family objects for designing rate-making system

Description

Current available distributions that can be used to designing of rate-making system .

Usage

```
## Default S3 method:
nlirms.family(object, ...)
nlirms.family(object, ...)
as.nlirms.family(object)
```

Arguments

object	a nlirms.family object
...	for further arguments

Details

there are several frequency distributions available to designing of rate-making system as follow:

Poissin-Gamma or Negative Binomial (PGA model)

Poissin-Inverse Gamma (PIGA model)

Poissin-Generalized Inverse Gaussian or Sichel (PGIG model)

Poissin-Inverse Gaussian (PGIG model reduce to the Poissin-Inverse Gaussian model for $\nu=-.5$)

Poissin-Harmonic (PGIG model reduce to the Poissin-Harmonic model for $\nu=0$)

there are several severity distributions available to designing of rate-making system as follow:

Exponential-Gamma (EGA model)

Exponential-Inverse Gamma or Pareto (EIGA model)

Exponential-Generalized Inverse Gaussian (EGIG model)

Exponential-Inverse Gaussian (EGIG model reduce to the Exponential-Inverse Gaussian model for $\nu=-.5$)

Exponential-Harmonic (EGIG model reduce to the Exponential-Harmonic model for $\nu=0$)

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References

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- Stasinopoulos, D. M., & Rigby, R. A. (2007). Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, 23(7), 1-46.

Examples

```
PGA(mu , sigma)
EGA(mu , sigma)
```

rmspfc

Rate-making system based on the posteriori frequency component

Description

rmspfc() function gives the rate-making system based on the posteriori frequency component. Values given by rmspfc() function is equal to with expected number of claims given by enc.family (i.e. enc.PGA, enc.PIGA, enc.PGIG) for different amounts of time and claim.

Usage

```
rmspfc(time = 5, claim = 5, fmu = .2, fsigma = 2, fnu = 1,
  family = "NO", round = 2, size = 8, padlength = 4, padwidth = 2,
  ...)
```

Arguments

time	time period to designing of rate-making system based on the posteriori frequency component
claim	number of claims to designing of rate-making system based on the posteriori frequency component

fmu	mu parameter of frequency model in designing of rate-making system
fsigma	sigma parameter of frequency model in designing of rate-making system
fnu	nu parameter of frequency model in designing of rate-making system
family	a nlrms.family object, which is used to define the frequency model to designing of rate-making system
round	rounds the rate-making system values to the specified number of decimal places
size	indicates the size of graphical table for rate-making system
padlength	indicates the length of each graphical table cells
padwidth	indicates the width of each graphical table cells
...	for further arguments

Details

rmspfc() function gives the rate-making system in the form of a table where each table cells is related to the one time and claim. for example the cell with time=2 and claim=1, shows the expected number of claims in next year for a ploicyholder that who had a one claim in past two years.

Value

rmspfc() function return the expected number of claims of policyholders based on the different models.

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References

- Frangos, N. E., & Vrontos, S. D. (2001). Design of optimal bonus-malus systems with a frequency and a severity component on an individual basis in automobile insurance. *ASTIN Bulletin: The Journal of the IAA*, 31(1), 1-22.
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- Stasinopoulos, D. M., & Rigby, R. A. (2007). Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, 23(7), 1-46.

Examples

```
# rate-Making system based on the Poisson-Gamma model for frequency component
rmspfc(time = 5, claim = 5, fmu = .2, fsigma = 2, fnu = 1, family = "PGA", round
= 2, size = 8, padlength = 4, padwidth = 2)

# rate-Making system based on the Poisson-Inverse Gamma model for frequency component
rmspfc(time = 5, claim = 5, fmu = .2, fsigma = 2, fnu = 1, family = "PIGA",
round = 2, size = 8, padlength = 4, padwidth = 2)

# rate-Making system based on the Poisson-Generalized Inverse Gaussian model for frequency
rmspfc(time = 5, claim = 5, fmu = .2, fsigma = 2, fnu = 1, family = "PGIG",
round = 2, size = 8, padlength = 4, padwidth = 2)
```

rmspfsc	<i>Rate-making system based on the posteriori frequency and severity component</i>
---------	--

Description

rmspfsc() function gives the rate-making system based on the posteriori frequency and severity component. Values given by rmspfc() function is equal to with expected number of claims given by enc.family (i.e. enc.PGA, enc.PIGA, enc.PGIG) multiplication in expected severity of claims given by esc.family (i.e. esc.EGA, esc.EIGA, esc.EGIG) for different amounts of time and claim.

Usage

```
rmspfsc(time=5 ,claim=5, fmu = .1, fsigma = 2, fnu = 1,
sumsev=100, smu = 50, ssigma = 3, snu = 2, family = list("NO","NO"),
round=2 ,size=8, padlength=4, padwidth=2, ...)
```

Arguments

time	time period to designing of rate-making system based on the posteriori frequency and severity component
claim	number of claims to designing of rate-making system based on the posteriori frequency and severity component
sumsev	sum severity of all claims to designing of rate-making system based on the posteriori frequency and severity component
fmu	mu parameter of frequency model in designing of rate-making system
fsigma	sigma parameter of frequency model in designing of rate-Making system
fnu	nu parameter of frequency model in designing of rate-making-system
smu	mu parameter of severity model in designing of rate-making-system
ssigma	sigma parameter of severity model in designing of rate-making system
snu	nu parameter of severity model in designing of rate-making system

family	a vector of nlirms.family's object, which first argument is used to define the frequency model and second argument is used to define the severity model to designing of rate-making system based on frequency and severity component.
round	rounds the rate-making system values to the specified number of decimal places
size	indicates the size of graphical table for rate-making system
padlength	indicates the length of each graphical table cells
padwidth	indicates the width of each graphical table cells
...	for further arguments

Details

rmspfsc() function gives the rate-making system in the form of a table where each table cells is related to the one time and claim. for example if sumsev=100, then the cell with time=3 claim=2, shows the pure premium in next year for a policyholder that who had a two claim in past three years in which the total size of two claim was equal to 100.

Value

rmspfsc() function return the expected fair premiums of policyholders based on the different models.

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- Frangos, N. E., & Vrontos, S. D. (2001). Design of optimal bonus-malus systems with a frequency and a severity component on an individual basis in automobile insurance. *ASTIN Bulletin: The Journal of the IAA*, 31(1), 1-22.
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Examples

```
# rate-making system based on the PGA model for frequency and EGA model for severity component
rmspfsc(time=5 ,claim=5, fmu = .1, fsigma = 2, fnu = 1, sumsev=100, smu = 50,
ssigma = 3, snu = 2, family = list("PGA","EGA"),round=2 ,size=8, padlength=4,
padwidth=2)
```

rmspsc

Rate-making system based on the posteriori severity component

Description

rmspsc() function gives the rate-making system based on the posteriori severity component. Values given by rmspsc() function is equal to with expected severity of claims given by esc.family (i.e. esc.EGA, esc.EIGA, esc.EGIG) for different amounts of claimS.

Usage

```
rmspsc(time=5, claim=5, sumsev=100, smu = 50, ssigma = 3, snu = 2,
family = "NO", round=2, size=8 , padlength=4, padwidth=2, ...)
```

Arguments

time	time period to designing of rate-making system based on the posteriori frequency component
claim	number of claims to designing of rate-making system based on the posteriori frequency component
sumsev	sum severity of all claims to designing of rate-making system based on the posteriori frequency component
smu	mu parameter of severity model in designing of rate-making system
ssigma	sigma parameter of severity model in designing of rate-making system
snu	nu parameter of severity model in designing of rate-making system
family	a nlirms.family object, which is used to define the severity model to designing of rate-making system
round	rounds the rate-making system values to the specified number of decimal places
size	indicates the size of graphical table for rate-making system
padlength	indicates the length of each graphical table cells
padwidth	indicates the width of each graphical table cells
...	for further arguments

Details

rmspfsc() function gives the rate-making system in the form of a table where each table cells is related to the one claim. for example if sumsev=100, then the cell with claim=2, shows the expected severity of claims in next year for a ploicyholder that who had a two claim in past years in which the total size of two claim was equal to 100. Rate-Making system based on the severity component does not dependent to the time.

Value

rmspsc() function return the expected severity of claims of policyholders based on the different models.

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Stasinopoulos, D. M., & Rigby, R. A. (2007). Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, 23(7), 1-46.

Examples

```
# rate-making system based on the Exponential-Gamma model for severity component
rmspsc(time=5, claim=5, sumsev=100, smu = 50, ssigma = 3, snu = 2, family
="EGA", round=2, size=8 , padlength=4, padwidth=2)
```

```
# rate-making system based on the Exponential-Inverse Gamma model for severity component
rmspsc(time=5, claim=5, sumsev=100, smu = 50, ssigma = 3, snu = 2, family
="EIGA", round=2, size=8 , padlength=4, padwidth=2)
```

```
# rate-making system based on the Exponential-Generalized Inverse Gaussian model for severity
rmspsc(time=5, claim=5, sumsev=100, smu = 50, ssigma = 3, snu = 2, family
="EGIG", round=2, size=8 , padlength=4, padwidth=2)
```

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