Package ‘spc’

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Title Statistical Process Control -- Calculation of ARL and Other Control Chart Performance Measures

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Depends R (>= 1.8.0)

Description Evaluation of control charts by means of
the zero-state, steady-state ARL (Average Run Length) and RL quantiles.
Setting up control charts for given in-control ARL. The control charts
under consideration are one- and two-sided EWMA, CUSUM, and
Shiryaev-Roberts schemes for monitoring the mean or variance of normally
distributed independent data. ARL calculation of the same set of schemes under drift (in the mean) are added.
Eventually, all ARL measures for the multivariate EWMA (MEWMA) are provided.

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NeedsCompilation yes

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R topics documented:

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dphat *Percent defective for normal samples*

Description

Density, distribution function and quantile function for the sample percent defective calculated on normal samples with mean equal to `mu` and standard deviation equal to `sigma`.

Usage

```
dphat(x, n, mu=0, sigma=1, type="known", LSL=-3, USL=3, nodes=30)

pphat(q, n, mu=0, sigma=1, type="known", LSL=-3, USL=3, nodes=30)

qphat(p, n, mu=0, sigma=1, type="known", LSL=-3, USL=3, nodes=30)
```

Arguments

`x, q`	vector of quantiles.
`p`	vector of probabilities.
`n`	sample size.
`mu, sigma`	parameters of the underlying normal distribution.
`type`	choose whether the standard deviation is given and fixed (`"known"`) or estimated and potententially monitored (`"estimated"`).
`LSL,USL`	lower and upper specification limit, respectively.
`nodes`	number of quadrature nodes needed for `type="estimated"`.

Details

Bruhn-Suhr/Krumbholz (1990) derived the cumulative distribution function of the sample percent defective calculated on normal samples to applying them for a new variables sampling plan. These results were heavily used in Krumbholz/Z\"oller (1995) for Shewhart and in Knoth/Steinmetz (2013) for EWMA control charts. For algorithmic details see, essentially, Bruhn-Suhr/Krumbholz (1990). Two design variants are treated: The simple case, `type="known"`, with known normal variance and the presumably much more relevant and considerably intricate case, `type="estimated"`, where both parameters of the normal distribution are unknown. Basically, given lower and upper specification limits and the normal distribution, one estimates the expected yield based on a normal sample of size `n`.

Value

Returns vector of pdf, cdf or qf values for the statistic phat.

Author(s)

Sven Knoth

References

M. Bruhn-Suhr and W. Krumbholz (1990), A new variables sampling plan for normally distributed lots with unknown standard deviation and double specification limits, *Statistical Papers* 31(1), 195-207.

W. Krumbholz and A. Z\"oller (1995), p-Karten vom Shewhartschen Typ f\"ur die messende Pr\"ufung, *Allgemeines Statistisches Archiv* 79, 347-360.

S. Knoth and S. Steinmetz (2013), EWMA p charts under sampling by variables, *International Journal of Production Research* 51(13), 3795-3807.

See Also

`phat.ewma.arl` for routines using the herewith considered phat statistic.

Examples

```
# Figures 1 (c) and (d) from Knoth/Steinmetz (2013)
n      <-  5
LSL    <- -3
USL    <-  3

par(mar=c(5, 5, 1, 1) + 0.1)

p.star <- 2*pnorm( (LSL-USL)/2 ) # for p <= p.star pdf and cdf vanish

p_ <- seq(p.star+1e-10, 0.07, 0.0001) # define support of Figure 1

# Figure 1 (c)
pp_ <- pphat(p_, n)
plot(p_, pp_, type="l", xlab="p", ylab=expression(P( hat(p) <= p )),
     xlim=c(0, 0.06), ylim=c(0,1), lwd=2)
abline(h=0:1, v=p.star, col="grey")

# Figure 1 (d)
dp_ <- dphat(p_, n)
plot(p_, dp_, type="l", xlab="p", ylab="f(p)", xlim=c(0, 0.06),
     ylim=c(0,50), lwd=2)
abline(h=0, v=p.star, col="grey")
```

`lns2ewma.arl` *Compute ARLs of EWMA ln $S\hat{}2$ control charts (variance charts)*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts (based on the log of the sample variance S^2) monitoring normal variance.

Usage

```
lns2ewma.arl(l,cl,cu,sigma,df,hs=NULL,sided="upper",r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`cl`	lower control limit of the EWMA control chart.
`cu`	upper control limit of the EWMA control chart.
`sigma`	true standard deviation.
`df`	actual degrees of freedom, corresponds to subsample size (for known mean it is equal to the subsample size, for unknown mean it is equal to subsample size minus one.
`hs`	so-called headstart (enables fast initial response) – the default value (hs=NULL) corresponds to the in-control mean of ln S^2.
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart with reflection at `cl`), `"lower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`r`	dimension of the resulting linear equation system: the larger the better.

Details

`lns2ewma.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

S. V. Crowder and M. D. Hamilton (1992), An EWMA for monitoring a process standard deviation, *Journal of Quality Technology 24*, 12-21.

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts for monitoring normal mean.

Examples

```
lns2ewma.ARL <- Vectorize("lns2ewma.arl", "sigma")

## Crowder/Hamilton (1992)
## moments of ln S^2
E_log_gamma <- function(df) log(2/df) + digamma(df/2)
V_log_gamma <- function(df) trigamma(df/2)
E_log_gamma_approx <- function(df) -1/df - 1/3/df^2 + 2/15/df^4
V_log_gamma_approx <- function(df) 2/df + 2/df^2 + 4/3/df^3 - 16/15/df^5

## results from Table 3 ( upper chart with reflection at 0 = log(sigma0=1) )
## original entries are (lambda = 0.05, K = 1.06, df=n-1=4)
# sigma   ARL
# 1       200
# 1.1      43
# 1.2      18
# 1.3      11
# 1.4       7.6
# 1.5       6.0
# 2         3.2

df <- 4
lambda <- .05
K <- 1.06
cu <- K * sqrt( lambda/(2-lambda) * V_log_gamma_approx(df) )

sigmas <- c(1 + (0:5)/10, 2)
arls <- round(lns2ewma.ARL(lambda, 0, cu, sigmas, df, hs=0, sided="upper"), digits=1)
data.frame(sigmas, arls)

## Knoth (2005)
## compare with Table 3 (p. 351)
lambda <- .05
df <- 4
K <- 1.05521
cu <- 1.05521 * sqrt( lambda/(2-lambda) * V_log_gamma_approx(df) )

## upper chart with reflection at sigma0=1 in Table 4
## original entries are
# sigma   ARL_0    ARL_-.267
# 1       200.0    200.0
# 1.1      43.04    41.55
# 1.2      18.10    19.92
# 1.3      10.75    13.11
# 1.4       7.63     9.93
# 1.5       5.97     8.11
# 2         3.17     4.67

M <- -0.267
cuM <- lns2ewma.crit(lambda, 200, df, cl=M, hs=M, r=60)[2]
arls1 <- round(lns2ewma.ARL(lambda, 0, cu, sigmas, df, hs=0, sided="upper"), digits=2)
arls2 <- round(lns2ewma.ARL(lambda, M, cuM, sigmas, df, hs=M, sided="upper", r=60), digits=2)
```

```
data.frame(sigmas, arls1, arls2)
```

---

lns2ewma.crit *Compute critical values of EWMA ln $S\hat{}2$ control charts (variance charts)*

Description

Computation of the critical values (similar to alarm limits) for different types of EWMA control charts (based on the log of the sample variance S^2) monitoring normal variance.

Usage

```
lns2ewma.crit(l,L0,df,sigma0=1,cl=NULL,cu=NULL,hs=NULL,sided="upper",mode="fixed",r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`L0`	in-control ARL.
`df`	actual degrees of freedom, corresponds to subsample size (for known mean it is equal to the subsample size, for unknown mean it is equal to subsample size minus one.
`sigma0`	in-control standard deviation.
`cl`	deployed for `sided="upper"`, that is, upper variance control chart with lower reflecting barrier `cl`.
`cu`	for two-sided (`sided="two"`) and fixed upper control limit (`mode="fixed"`), for all other cases `cu` is ignored.
`hs`	so-called headstart (enables fast initial response) – the default value (hs=NULL) corresponds to the in-control mean of ln S^2.
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart with reflection at `cl`), `"lower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`mode`	only deployed for `sided="two"` – with `"fixed"` an upper control limit (see `cu`) is set and only the lower is calculated to obtain the in-control ARL `L0`, while with `"unbiased"` a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated). With `"vanilla"` limits symmetric around the in-control mean of ln S^2 are determined, while for `"eq.tails"` the in-control ARL values of two single EWMA variance charts (decompose the two-sided scheme into one lower and one upper scheme) are matched.
`r`	dimension of the resulting linear equation system: the larger the more accurate.

Details

`lns2ewma.crit` determines the critical values (similar to alarm limits) for given in-control ARL `L0` by applying secant rule and using `lns2ewma.arl()`. In case of `sided="two"` and `mode="unbiased"` a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at `sigma0`. See Knoth (2010) and the related example.

Value

Returns the lower and upper control limit `cl` and `cu`.

Author(s)

Sven Knoth

References

C. A. Acosta-Mej\'ia and J. J. Pignatiello Jr. and B. V. Rao (1999), A comparison of control charting procedures for monitoring process dispersion, *IIE Transactions 31*, 569-579.

S. V. Crowder and M. D. Hamilton (1992), An EWMA for monitoring a process standard deviation, *Journal of Quality Technology 24*, 12-21.

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2010), Control Charting Normal Variance – Reflections, Curiosities, and Recommendations, in *Frontiers in Statistical Quality Control 9*, H.-J. Lenz and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 3-18.

See Also

`lns2ewma.arl` for calculation of ARL of EWMA ln S^2 control charts.

Examples

```
## Knoth (2005)
## compare with Table 3 (p. 351)
L0 <- 200
l <- .05
df <- 4
limits <- lns2ewma.crit(l, L0, df, cl=0, hs=0)
limits["cu"]
```

mewma.arl *Compute ARLs of MEWMA control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) for multivariate exponentially weighted moving average (MEWMA) charts monitoring multivariate normal mean.

Usage

```
mewma.arl(l, cE, p, delta=0, hs=0, r=20, ntype=NULL, qm0=20, qm1=qm0)

mewma.arl.f(l, cE, p, delta=0, r=20, ntype=NULL, qm0=20, qm1=qm0)

mewma.ad(l, cE, p, delta=0, r=20, n=20, type="cond", hs=0, ntype=NULL, qm0=20, qm1=qm0)
```

Arguments

`l`	smoothing parameter lambda of the MEWMA control chart.
`cE`	alarm threshold of the MEWMA control chart.
`p`	dimension of multivariate normal distribution.
`delta`	magnitude of the potential change, `delta=0` refers to the in-control state.
`hs`	so-called headstart (enables fast initial response) – must be non-negative.
`r`	number of quadrature nodes – dimension of the resulting linear equation system for `delta` = 0. For non-zero `delta` this dimension is mostly r^2 (Markov chain approximation leads to some larger values). Caution: If `ntype` is set to `"co"` (collocation), then values of `r` larger than 20 lead to large computing times. For the other selections this would happen for values larger than 40.
`ntype`	choose the numerical algorithm to solve the ARL integral equation. For `delta`=0: Possible values are `"gl"`, `"gl2"` (gauss-legendre, classic and with variables change: square), `"co"` (collocation, for `delta` > 0 with sin transformation), `"ra"` (radau), `"cc"` (clenshaw-curtis), `"mc"` (markov chain), and `"sr"` (simpson rule). For `delta` larger than 0, some more values besides the others are possible: `"gl3"`, `"gl4"`, `"gl5"` (gauss-legendre with a further change in variables: sin, tan, sinh), `"co2"`, `"co3"` (collocation with some trimming and tan as quadrature stabilizing transformations, respectively). If it is set to `NULL` (the default), then for `delta`=0 then `"gl2"` is chosen. If `delta` larger than 0, then for `p` equal 2 or 4 `"gl3"` and for all other values `"gl5"` is taken. `"ra"` denotes the method used in Rigdon (1995a). `"mc"` denotes the Markov chain approximation.
`type`	switch between `"cond"` and `"cycl"` for differentiating between the conditional (no false alarm) and the cyclical (after false alarm re-start in `hs`), respectively.
`n`	number of quadrature nodes for Calculating the steady-state ARL integral(s).
`qm0,qm1`	number of collocation quadrature nodes for the out-of-control case (`qm0` for the inner integral, `qm1` for the outer one), that is, for positive `delta`, and for the in-control case (now only `qm0` is deployed) if via `ntype` the collocation procedure is requested.

Details

Basically, this is the implementation of different numerical algorithms for solving the integral equation for the MEWMA in-control (`delta` = 0) ARL introduced in Rigdon (1995a) and out-of-control (`delta` != 0) ARL in Rigdon (1995b). Most of them are nothing else than the Nystroem approach – the integral is replaced by a suitable quadrature. Here, the Gauss-Legendre (more powerful), Radau (used by Rigdon, 1995a), Clenshaw-Curtis, and Simpson rule (which is really bad) are provided. Additionally, the collocation approach is offered as well, because it is much better for small odd values for `p`. FORTRAN code for the Radau quadrature based Nystroem of Rigdon (1995a) was published in Bodden and Rigdon (1999) – see also `http://lib.stat.cmu.edu/jqt/31-1`. Furthermore, FORTRAN code for the Markov chain approximation (in- and out-ot-control) could be found at

http://lib.stat.cmu.edu/jqt/33-4. The related papers are Runger and Prabhu (1996) and Molnau et al. (2001). The idea of the Clenshaw-Curtis quadrature was taken from Capizzi and Masarotto (2010), who successfully deployed a modified Clenshaw-Curtis quadrature to calculate the ARL of combined (univariate) Shewhart-EWMA charts. It turns out that it works also nicely for the MEWMA ARL. The version `mewma.arl.f()` without the argument `hs` provides the ARL as function of one (in-control) or two (out-of-control) arguments.

Value

Returns a single value which is simply the zero-state ARL.

Author(s)

Sven Knoth

References

Kevin M. Bodden and Steven E. Rigdon (1999), A program for approximating the in-control ARL for the MEWMA chart, *Journal of Quality Technology 31(1)*, 120-123.

Giovanna Capizzi and Guido Masarotto (2010), Evaluation of the run-length distribution for a combined Shewhart-EWMA control chart, *Statistics and Computing 20(1)*, 23-33.

Sven Knoth (2017), ARL Numerics for MEWMA Charts, *Journal of Quality Technology 49(1)*, 78-89.

Wade E. Molnau et al. (2001), A Program for ARL Calculation for Multivariate EWMA Charts, *Journal of Quality Technology 33(4)*, 515-521.

Sharad S. Prabhu and George C. Runger (1997), Designing a multivariate EWMA control chart, *Journal of Quality Technology 29(1)*, 8-15.

Steven E. Rigdon (1995a), An integral equation for the in-control average run length of a multivariate exponentially weighted moving average control chart, *J. Stat. Comput. Simulation 52(4)*, 351-365.

Steven E. Rigdon (1995b), A double-integral equation for the average run length of a multivariate exponentially weighted moving average control chart, *Stat. Probab. Lett. 24(4)*, 365-373.

George C. Runger and Sharad S. Prabhu (1996), A Markov Chain Model for the Multivariate Exponentially Weighted Moving Averages Control Chart, *J. Amer. Statist. Assoc. 91(436)*, 1701-1706.

See Also

`mewma.crit` for getting the alarm threshold to attain a certain in-control ARL.

Examples

```
# Rigdon (1995a), p. 357, Tab. 1
p <- 2
r <- 0.25
h4 <- c(8.37, 9.90, 11.89, 13.36, 14.82, 16.72)
for ( i in 1:length(h4) ) cat(paste(h4[i], "\t", round(mewma.arl(r, h4[i], p, ntype="ra")), "\n"))

r <- 0.1
h4 <- c(6.98, 8.63, 10.77, 12.37, 13.88, 15.88)
for ( i in 1:length(h4) ) cat(paste(h4[i], "\t", round(mewma.arl(r, h4[i], p, ntype="ra")), "\n"))


# Rigdon (1995b), p. 372, Tab. 1
## Not run:
r <- 0.1
p <- 4
h <- 12.73
for ( sdelta in c(0, 0.125, 0.25, .5, 1, 2, 3) )
  cat(paste(sdelta, "\t",
      round(mewma.arl(r, h, p, delta=sdelta^2, ntype="ra", r=25), digits=2), "\n"))

p <- 5
h <- 14.56
for ( sdelta in c(0, 0.125, 0.25, .5, 1, 2, 3) )
  cat(paste(sdelta, "\t",
      round(mewma.arl(r, h, p, delta=sdelta^2, ntype="ra", r=25), digits=2), "\n"))

p <- 10
h <- 22.67
for ( sdelta in c(0, 0.125, 0.25, .5, 1, 2, 3) )
  cat(paste(sdelta, "\t",
      round(mewma.arl(r, h, p, delta=sdelta^2, ntype="ra", r=25), digits=2), "\n"))

## End(Not run)

# Runger/Prabhu (1996), p. 1704, Tab. 1
## Not run:
r <- 0.1
p <- 4
H <- 12.73
cat(paste(0, "\t", round(mewma.arl(r, H, p, delta=0, ntype="mc", r=50), digits=2), "\n"))
for ( delta in c(.5, 1, 1.5, 2, 3) )
  cat(paste(delta, "\t",
      round(mewma.arl(r, H, p, delta=delta, ntype="mc", r=25), digits=2), "\n"))
# compare with Fortran program (MEWMA-ARLs.f90) from Molnau et al. (2001) with m1 = m2 = 25
# H4      P     R   DEL  ARL
# 12.73  4.  0.10  0.00 199.78
# 12.73  4.  0.10  0.50  35.05
```

```
# 12.73  4.  0.10  1.00  12.17
# 12.73  4.  0.10  1.50   7.22
# 12.73  4.  0.10  2.00   5.19
# 12.73  4.  0.10  3.00   3.42

p <- 20
H <- 37.01
cat(paste(0, "\t",
    round(mewma.arl(r, H, p, delta=0, ntype="mc", r=50), digits=2), "\n"))
for ( delta in c(.5, 1, 1.5, 2, 3) )
  cat(paste(delta, "\t",
      round(mewma.arl(r, H, p, delta=delta, ntype="mc", r=25), digits=2), "\n"))
# compare with Fortran program (MEWMA-ARLs.f90) from Molnau et al. (2001) with m1 = m2 = 25
# H4      P     R   DEL  ARL
# 37.01 20.  0.10  0.00 199.09
# 37.01 20.  0.10  0.50  61.62
# 37.01 20.  0.10  1.00  20.17
# 37.01 20.  0.10  1.50  11.40
# 37.01 20.  0.10  2.00   8.03
# 37.01 20.  0.10  3.00   5.18

## End(Not run)

# Knoth (2017), p. 85, Tab. 3, rows with p=3
## Not run:
p <- 3
lambda <- 0.05
h4 <- mewma.crit(lambda, 200, p)
benchmark <- mewma.arl(lambda, h4, p, delta=1, r=50)

mc.arl  <- mewma.arl(lambda, h4, p, delta=1, r=25, ntype="mc")
ra.arl  <- mewma.arl(lambda, h4, p, delta=1, r=27, ntype="ra")
co.arl  <- mewma.arl(lambda, h4, p, delta=1, r=12, ntype="co2")
gl3.arl <- mewma.arl(lambda, h4, p, delta=1, r=30, ntype="gl3")
gl5.arl <- mewma.arl(lambda, h4, p, delta=1, r=25, ntype="gl5")

abs( benchmark - data.frame(mc.arl, ra.arl, co.arl, gl3.arl, gl5.arl) )

## End(Not run)

# Prabhu/Runger (1997), p. 13, Tab. 3
## Not run:
p <- 2
r <- 0.1
H <- 8.64
cat(paste(0, "\t",
    round(mewma.ad(r, H, p, delta=0, type="cycl", ntype="mc", r=60), digits=2), "\n"))
for ( delta in c(.5, 1, 1.5, 2, 3) )
  cat(paste(delta, "\t",
     round(mewma.ad(r, H, p, delta=delta, type="cycl", ntype="mc", r=30), digits=2), "\n"))

# better accuracy
for ( delta in c(0, .5, 1, 1.5, 2, 3) )
```

```
  cat(paste(delta, "\t",
      round(mewma.ad(r, H, p, delta=delta^2, type="cycl", r=30), digits=2), "\n"))

## End(Not run)
```

---

mewma.crit *Compute alarm threshold of MEWMA control charts*

Description

Computation of the alarm threshold for multivariate exponentially weighted moving average (MEWMA) charts monitoring multivariate normal mean.

Usage

```
mewma.crit(l, L0, p, hs=0, r=20)
```

Arguments

`l`	smoothing parameter lambda of the MEWMA control chart.
`L0`	in-control ARL.
`p`	dimension of multivariate normal distribution.
`hs`	so-called headstart (enables fast initial response) – must be non-negative.
`r`	number of quadrature nodes – dimension of the resulting linear equation system.

Details

`mewma.crit` determines the alarm threshold of for given in-control ARL `L0` by applying secant rule and using `mewma.arl()` with `ntype="gl2"`.

Value

Returns a single value which resembles the critical value `c`.

Author(s)

Sven Knoth

References

Sven Knoth (2017), ARL Numerics for MEWMA Charts, *Journal of Quality Technology 49(1)*, 78-89.

Steven E. Rigdon (1995), An integral equation for the in-control average run length of a multivariate exponentially weighted moving average control chart, *J. Stat. Comput. Simulation 52(4)*, 351-365.

See Also

`mewma.arl` for zero-state ARL computation.

Examples

```
# Rigdon (1995), p. 358, Tab. 1
p <- 4
L0 <- 500
r <- .25
h4 <- mewma.crit(r, L0, p)
h4
## original value is 16.38.

# Knoth (2017), p. 82, Tab. 2
p <- 3
L0 <- 1e3
lambda <- c(0.25, 0.2, 0.15, 0.1, 0.05)
h4 <- rep(NA, length(lambda) )
for ( i in 1:length(lambda) ) h4[i] <- mewma.crit(lambda[i], L0, p, r=20)
round(h4, digits=2)
## original values are
## 15.82 15.62 15.31 14.76 13.60
```

---

mewma.psi *Compute steady-state density of the MEWMA statistic*

Description

Computation of the (zero-state) steady-state density function of the statistic deployed in multivariate exponentially weighted moving average (MEWMA) charts monitoring multivariate normal mean.

Usage

```
mewma.psi(l, cE, p, type="cond", hs=0, r=20)
```

Arguments

`l`	smoothing parameter lambda of the MEWMA control chart.
`cE`	alarm threshold of the MEWMA control chart.
`p`	dimension of multivariate normal distribution.
`type`	switch between "cond" and "cycl" for differentiating between the conditional (no false alarm) and the cyclical (after false alarm re-start in `hs`), respectively.
`hs`	the re-starting point for the cyclical steady-state framework.
`r`	number of quadrature nodes.

Details

Basically, ideas from Knoth (2017, MEWMA numerics) and Knoth (2016, steady-state ARL concepts) are merged. More details will follow.

Value

Returns a function.

Author(s)

Sven Knoth

References

Sven Knoth (2016), The Case Against the Use of Synthetic Control Charts, *Journal of Quality Technology 48(2)*, 178-195.

Sven Knoth (2017), ARL Numerics for MEWMA Charts, *Journal of Quality Technology 49(1)*, 78-89.

Sven Knoth (2018), The Steady-State Behavior of Multivariate Exponentially Weighted Moving Average Control Charts, unpublished manuscript.

See Also

`mewma.arl` for calculating the in-control ARL of MEWMA.

Examples

```
lambda <- 0.1
L0 <- 1000
p <- 3
h4 <- mewma.crit(lambda, L0, p)
x_ <- seq(0, h4*lambda/(2-lambda), by=0.002)
psi <- mewma.psi(lambda, h4, p)
psi_ <- psi(x_)
#plot(x_, psi_, type="l", xlab="x", ylab=expression(psi(x)))
```

`p.ewma.arl` *Compute ARLs of binomial EWMA p control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) at given rate `p`.

Usage

```
p.ewma.arl(lambda, ucl, n, p, z0, d.res=1, r.mode="ieee.round", i.mode="integer")
```

Arguments

`lambda`	smoothing parameter of the EWMA p control chart.
`ucl`	upper control limit of the EWMA p control chart.
`n`	subgroup size.
`p`	(failure/success) rate.
`z0`	so-called headstart (give fast initial response).
`d.res`	resolution (see details).
`r.mode`	round mode – allowed modes are `"gan.floor"`, `"floor"`, `"ceil"`, `"ieee.round"`, `"round"`, `"mix"`.
`i.mode`	type of interval center – `"integer"` or `"half"` integer.

Details

The monitored data follow a binomial distribution with size `n` and failure/success probability `p`. The ARL values of the resulting EWMA control chart are determined by Markov chain approximation. Here, the original EWMA values are approximated by multiples of one over `d.res`. Different ways of rounding (see `r.mode`) to the next multiple are implemented. Besides Gan's paper nothing is published about the numerical subtleties.

Value

Return single value which resemble the ARL.

Author(s)

Sven Knoth

References

F. F. Gan (1990), Monitoring observations generated from a binomial distribution using modified exponentially weighted moving average control chart, *J. Stat. Comput. Simulation* 37, 45-60.

S. Knoth and S. Steinmetz (2013), EWMA `p` charts under sampling by variables, *International Journal of Production Research* 51, 3795-3807.

See Also

later.

Examples

```
## Gan (1990)

# Table 1

n <- 150
p0 <- .1
z0 <- n*p0
```

```
lambda <- c(1, .51, .165)
hu <- c(27, 22, 18)

p.value <- .1 + (0:20)/200

p.EWMA.arl <- Vectorize(p.ewma.arl, "p")

arl1.value <- round(p.EWMA.arl(lambda[1], hu[1], n, p.value, z0, r.mode="round"), digits=2)
arl2.value <- round(p.EWMA.arl(lambda[2], hu[2], n, p.value, z0, r.mode="round"), digits=2)
arl3.value <- round(p.EWMA.arl(lambda[3], hu[3], n, p.value, z0, r.mode="round"), digits=2)

arls <- matrix(c(arl1.value, arl2.value, arl3.value), ncol=length(lambda))
rownames(arls) <- p.value
colnames(arls) <- paste("lambda =", lambda)
arls

## Knoth/Steinmetz (2013)

n <- 5
p0 <- 0.02
z0 <- n*p0
lambda <- 0.3
ucl <- 0.649169922 ## in-control ARL 370.4 (determined with d.res = 2^14 = 16384)

res.list <- 2^(1:12)
arl.list <- NULL
for ( res in res.list ) {
  arl <- p.ewma.arl(lambda, ucl, n, p0, z0, d.res=res)
  arl.list <- c(arl.list, arl)
}
cbind(res.list, arl.list)
```

---

phat.ewma.arl *Compute ARLs of EWMA phat control charts*

Description

Computation of the (zero-state) Average Run Length (ARL), upper control limit (ucl) for given in-control ARL, and lambda for minimal out-of control ARL at given shift.

Usage

```
phat.ewma.arl(lambda, ucl, mu, n, z0, sigma=1, type="known", LSL=-3, USL=3, N=15,
qm=25, ntype="coll")

phat.ewma.crit(lambda, L0, mu, n, z0, sigma=1, type="known", LSL=-3, USL=3, N=15, qm=25)

phat.ewma.lambda(L0, mu, n, z0, sigma=1, type="known", max_l=1, min_l=.001, LSL=-3, USL=3,
qm=25)
```

Arguments

`lambda`	smoothing parameter of the EWMA control chart.
`ucl`	upper control limit of the EWMA phat control chart.
`L0`	pre-defined in-control ARL (Average Run Length).
`mu`	true mean or mean where the ARL should be minimized (then the in-control mean is simply 0).
`n`	subgroup size.
`z0`	so-called headstart (gives fast initial response).
`type`	choose whether the standard deviation is given and fixed (`"known"`) or estimated and potentially monitored (`"estimated"`).
`sigma`	actual standard deviation of the data – the in-control value is 1.
`max_l, min_l`	maximal and minimal value for optimal lambda search.
`LSL,USL`	lower and upper specification limit, respectively.
`N`	size of collocation base, dimension of the resulting linear equation system is equal to `N`.
`qm`	number of nodes for collocation quadratures.
`ntype`	switch between the default method `coll` (collocation) and the classic one `markov` (Markov chain approximation) for calculating the ARL numerically.

Details

The three implemented functions allow to apply a new type control chart. Basically, lower and upper specification limits are given. The monitoring vehicle then is the empirical probability that an item will not follow these specification given the sequence of sample means. If the related EWMA sequence violates the control limits, then the alarm indicates a significant process deterioration. For details see the paper mentioned in the references. To be able to construct the control charts, see the first example.

Value

Return single values which resemble the ARL, the critical value, and the optimal lambda, respectively.

Author(s)

Sven Knoth

References

S. Knoth and S. Steinmetz (2013), EWMA `p` charts under sampling by variables, *International Journal of Production Research* 51, 3795-3807.

See Also

`sewma.arl` for a further collocation based ARL calculation routine.

Examples

```
## Simple example to demonstrate the chart.

# some functions
h.mu <- function(mu) pnorm(LSL-mu) + pnorm(mu-USL)
ewma <- function(x, lambda=0.1, z0=0) filter(lambda*x, 1-lambda, m="r", init=z0)

# parameters
LSL <- -3       # lower specification limit
USL <-  3 # upper specification limit
n <- 5 # batch size
lambda <- 0.1 # EWMA smoothing parameter
L0 <- 1000 # in-control Average Run Length (ARL)
z0 <- h.mu(0) # start at minimal defect level
ucl <- phat.ewma.crit(lambda, L0, 0, n, z0, LSL=LSL, USL=USL)

# data
x0 <- matrix(rnorm(50*n), ncol=5) # in-control data
x1 <- matrix(rnorm(50*n, mean=0.5), ncol=5)# out-of-control data
x <- rbind(x0,x1) # all data

# create chart
xbar <- apply(x, 1, mean)
phat <- h.mu(xbar)
z <- ewma(phat, lambda=lambda, z0=z0)
plot(1:length(z), z, type="l", xlab="batch", ylim=c(0,.02))
abline(h=z0, col="grey", lwd=.7)
abline(h=ucl, col="red")


## S. Knoth, S. Steinmetz (2013)

# Table 1

lambdas <- c(.5, .25, .2, .1)
L0 <- 370.4
n <- 5
LSL <- -3
USL <- 3

phat.ewma.CRIT <- Vectorize("phat.ewma.crit", "lambda")
p.star <- pnorm( LSL ) + pnorm( -USL ) ## lower bound of the chart
ucls <- phat.ewma.CRIT(lambdas, L0, 0, n, p.star, LSL=LSL, USL=USL)
print(cbind(lambdas, ucls))

# Table 2

mus <- c((0:4)/4, 1.5, 2, 3)
phat.ewma.ARL <- Vectorize("phat.ewma.arl", "mu")
arls <- NULL
for ( i in 1:length(lambdas) ) {
  arls <- cbind(arls, round(phat.ewma.ARL(lambdas[i], ucls[i], mus,
```

```
                n, p.star, LSL=LSL, USL=USL), digits=2))
}
arls <- data.frame(arls, row.names=NULL)
names(arls) <- lambdas
print(arls)

# Table 3

## Not run:
mus <- c(.25, .5, 1, 2)
phat.ewma.LAMBDA <- Vectorize("phat.ewma.lambda", "mu")
lambdas <- phat.ewma.LAMBDA(L0, mus, n, p.star, LSL=LSL, USL=USL)
print(cbind(mus, lambdas))
## End(Not run)
```

`quadrature.nodes.weights`

Calculate quadrature nodes and weights

Description

Computation of the nodes and weights to enable numerical quadrature.

Usage

```
quadrature.nodes.weights(n, type="GL", x1=-1, x2=1)
```

Arguments

`n`	number of nodes (and weights).
`type`	quadrature type – currently Gauss-Legendre, `"GL"`, and Radau, `"Ra"`, are supported.
`x1`	lower limit of the integration interval.
`x2`	upper limit of the integration interval.

Details

A more detailed description will follow soon. The algorithm for the Gauss-Legendre quadrature was delivered by Knut Petras to me, while the one for the Radau quadrature was taken from John Burkardt.

Value

Returns two vectors which hold the needed quadrature nodes and weights.

Author(s)

Sven Knoth

References

H. Brass and K. Petras (2011), *Quadrature Theory. The Theory of Numerical Integration on a Compact Interval,* Mathematical Surveys and Monographs, American Mathematical Society.

See Also

Many of the ARL routines use the Gauss-Legendre nodes.

Examples

```
# GL
n <- 10
qnw <-quadrature.nodes.weights(n, type="GL")
qnw

# Radau
n <- 10
qnw <-quadrature.nodes.weights(n, type="Ra")
qnw
```

`scusum.arl` *Compute ARLs of CUSUM control charts (variance charts)*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM control charts (based on the sample variance S^2) monitoring normal variance.

Usage

```
scusum.arl(k, h, sigma, df, hs=0, sided="upper", k2=NULL,
h2=NULL, hs2=0, r=40, qm=30, version=2)
```

Arguments

`k`	reference value of the CUSUM control chart.
`h`	decision interval (alarm limit, threshold) of the CUSUM control chart.
`sigma`	true standard deviation.
`df`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided CUSUM-S^2 control charts by choosing `"upper"` (upper chart), `"lower"` (lower chart), and `"two"` (two-sided chart), respectively. Note that for the two-sided chart the parameters `"k2"` and `"h2"` have to be set too.

k2	In case of a two-sided CUSUM chart for variance the reference value of the lower chart.
h2	In case of a two-sided CUSUM chart for variance the decision interval of the lower chart.
hs2	In case of a two-sided CUSUM chart for variance the headstart of the lower chart.
r	Dimension of the resulting linear equation system (highest order of the collocation polynomials times number of intervals – see Knoth 2006).
qm	Number of quadrature nodes for calculating the collocation definite integrals.
version	Distinguish version numbers (1,2,...). For internal use only.

Details

`scusum.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of collocation (piecewise Chebyshev polynomials).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2006), Computation of the ARL for CUSUM-S^2 schemes, *Computational Statistics & Data Analysis 51*, 499-512.

See Also

`xcusum.arl` for zero-state ARL computation of CUSUM control charts for monitoring normal mean.

Examples

```
## Knoth (2006)
## compare with Table 1 (p. 507)
k <- 1.46 # sigma1 = 1.5
df <- 1
h <- 10

# original values
# sigma coll63       BE     Hawkins  MC 10^9 (s.e.)
# 1     260.7369  260.7546  261.32  260.7399 (0.0081)
# 1.1    90.1319   90.1389   90.31   90.1319 (0.0027)
# 1.2    43.6867   43.6897   43.75   43.6845 (0.0013)
```

```
# 1.3    26.2916   26.2932   26.32   26.2929 (0.0007)
# 1.4    18.1231   18.1239   18.14   18.1235 (0.0005)
# 1.5    13.6268   13.6273   13.64   13.6272 (0.0003)
# 2       5.9904    5.9910    5.99    5.9903 (0.0001)
# replicate the column coll63
sigma <- c(1, 1.1, 1.2, 1.3, 1.4, 1.5, 2)
arl <- rep(NA, length(sigma))
for ( i in 1:length(sigma) )
  arl[i] <- round(scusum.arl(k, h, sigma[i], df, r=63, qm=20, version=2), digits=4)
data.frame(sigma, arl)
```

`scusum.crit` *Compute decision intervals of CUSUM control charts (variance charts)*

Description

omputation of the decision intervals (alarm limits) for different types of CUSUM control charts (based on the sample variance S^2) monitoring normal variance.

Usage

```
scusum.crit(k, L0, sigma, df, hs=0, sided="upper", mode="eq.tails",
k2=NULL, hs2=0, r=40, qm=30)
```

Arguments

`k`	reference value of the CUSUM control chart.
`L0`	in-control ARL.
`sigma`	true standard deviation.
`df`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided CUSUM-S^2 control charts by choosing `"upper"` (upper chart), `"lower"` (lower chart), and `"two"` (two-sided chart), respectively. Note that for the two-sided chart the parameters `"k2"` and `"h2"` have to be set too.
`mode`	only deployed for `sided="two"` – with `"eq.tails"` two one-sided CUSUM charts (lower and upper) with the same in-control ARL are coupled. With `"unbiased"` a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
`k2`	in case of a two-sided CUSUM chart for variance the reference value of the lower chart.
`hs2`	in case of a two-sided CUSUM chart for variance the headstart of the lower chart.

`r`	Dimension of the resulting linear equation system (highest order of the collocation polynomials times number of intervals – see Knoth 2006).
`qm`	Number of quadrature nodes for calculating the collocation definite integrals.

Details

`scusum.crit` ddetermines the decision interval (alarm limit) for given in-control ARL `L0` by applying secant rule and using `scusum.arl()`.

Value

Returns a single value which resembles the decision interval `h`.

Author(s)

Sven Knoth

References

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2006), Computation of the ARL for CUSUM-S^2 schemes, *Computational Statistics & Data Analysis 51*, 499-512.

See Also

`xcusum.arl` for zero-state ARL computation of CUSUM control charts monitoring normal mean.

Examples

```
## Knoth (2006)
## compare with Table 1 (p. 507)
k <- 1.46 # sigma1 = 1.5
df <- 1
L0 <- 260.74
h <- scusum.crit(k, L0, 1, df)
h
# original value is 10
```

`scusums.arl` *Compute ARLs of CUSUM-Shewhart control charts (variance charts)*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM-Shewhart combo control charts (based on the sample variance S^2) monitoring normal variance.

Usage

```
scusums.arl(k, h, cS, sigma, df, hs=0, sided="upper", k2=NULL,
h2=NULL, hs2=0, r=40, qm=30, version=2)
```

Arguments

`k`	reference value of the CUSUM control chart.
`h`	decision interval (alarm limit, threshold) of the CUSUM control chart.
`cS`	Shewhart limit.
`sigma`	true standard deviation.
`df`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided CUSUM-S^2 control charts by choosing `"upper"` (upper chart), `"lower"` (lower chart), and `"two"` (two-sided chart), respectively. Note that for the two-sided chart the parameters `"k2"` and `"h2"` have to be set too.
`k2`	In case of a two-sided CUSUM chart for variance the reference value of the lower chart.
`h2`	In case of a two-sided CUSUM chart for variance the decision interval of the lower chart.
`hs2`	In case of a two-sided CUSUM chart for variance the headstart of the lower chart.
`r`	Dimension of the resulting linear equation system (highest order of the collocation polynomials times number of intervals – see Knoth 2006).
`qm`	Number of quadrature nodes for calculating the collocation definite integrals.
`version`	Distinguish version numbers (1,2,...). For internal use only.

Details

`scusums.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of collocation (piecewise Chebyshev polynomials).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

S. Knoth (2006), Computation of the ARL for CUSUM-S^2 schemes, *Computational Statistics & Data Analysis 51*, 499-512.

See Also

scusum.arl for zero-state ARL computation of standalone CUSUM control charts for monitoring normal variance.

Examples

```
## will follow
```

---

sewma.arl *Compute ARLs of EWMA control charts (variance charts)*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts (based on the sample variance S^2) monitoring normal variance.

Usage

```
sewma.arl(l,cl,cu,sigma,df,s2.on=TRUE,hs=NULL,sided="upper",r=40,qm=30)
```

Arguments

l	smoothing parameter lambda of the EWMA control chart.
cl	lower control limit of the EWMA control chart.
cu	upper control limit of the EWMA control chart.
sigma	true standard deviation.
df	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
s2.on	distinguishes between S^2 and S chart.
hs	so-called headstart (enables fast initial response); the default (NULL) yields the expected in-control value of S^2 (1) and S (c_4), respectively.
sided	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing "upper" (upper chart without reflection at cl – the actual value of cl is not used), "Rupper" (upper chart with reflection at cl), "Rlower" (lower chart with reflection at cu), and "two" (two-sided chart), respectively.
r	dimension of the resulting linear equation system (highest order of the collocation polynomials).
qm	number of quadrature nodes for calculating the collocation definite integrals.

Details

sewma.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of collocation (Chebyshev polynomials).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2006), Computation of the ARL for CUSUM-S^2 schemes, *Computational Statistics & Data Analysis 51*, 499-512.

See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts for monitoring normal mean.

Examples

```
## Knoth (2005)
## compare with Table 1 (p. 347): 249.9997
## Monte Carlo with 10^9 replicates: 249.9892 +/- 0.008
l <- .025
df <- 1
cu <- 1 + 1.661865*sqrt(l/(2-l))*sqrt(2/df)
sewma.arl(l,0,cu,1,df)

## ARL values for upper and lower EWMA charts with reflecting barriers
## (reflection at in-control level sigma0 = 1)
## examples from Knoth (2006), Tables 4 and 5

Ssewma.arl <- Vectorize("sewma.arl", "sigma")

## upper chart with reflection at sigma0=1 in Table 4
## original entries are
# sigma   ARL
# 1       100.0
# 1.01    85.3
# 1.02    73.4
# 1.03    63.5
# 1.04    55.4
# 1.05    48.7
# 1.1     27.9
# 1.2     12.9
# 1.3     7.86
# 1.4     5.57
# 1.5     4.30
# 2       2.11

## Not run:
```

```
l <- 0.15
df <- 4
cu <- 1 + 2.4831*sqrt(l/(2-l))*sqrt(2/df)
sigmas <- c(1 + (0:5)/100, 1 + (1:5)/10, 2)
arls <- round(Ssewma.arl(l, 1, cu, sigmas, df, sided="Rupper", r=100), digits=2)
data.frame(sigmas, arls)
## End(Not run)

## lower chart with reflection at sigma0=1 in Table 5
## original entries are
# sigma   ARL
# 1       200.04
# 0.9     38.47
# 0.8     14.63
# 0.7     8.65
# 0.6     6.31

## Not run:
l <- 0.115
df <- 5
cl <- 1 - 2.0613*sqrt(l/(2-l))*sqrt(2/df)
sigmas <- c((10:6)/10)
arls <- round(Ssewma.arl(l, cl, 1, sigmas, df, sided="Rlower", r=100), digits=2)
data.frame(sigmas, arls)
## End(Not run)
```

`sewma.arl.prerun` *Compute ARLs of EWMA control charts (variance charts) in case of estimated parameters*

Description

Computation of the (zero-state) Average Run Length (ARL) for EWMA control charts (based on the sample variance S^2) monitoring normal variance with estimated parameters.

Usage

```
sewma.arl.prerun(l, cl, cu, sigma, df1, df2, hs=1, sided="upper",
r=40, qm=30, qm.sigma=30, truncate=1e-10)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`cl`	lower control limit of the EWMA control chart.
`cu`	upper control limit of the EWMA control chart.
`sigma`	true standard deviation.
`df1`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.

`df2`	degrees of freedom of the pre-run variance estimator.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`),`"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`r`	dimension of the resulting linear equation system (highest order of the collocation polynomials).
`qm`	number of quadrature nodes for calculating the collocation definite integrals.
`qm.sigma`	number of quadrature nodes for convoluting the standard deviation uncertainty.
`truncate`	size of truncated tail.

Details

Essentially, the ARL function `sewma.arl` is convoluted with the distribution of the sample standard deviation. For details see Jones/Champ/Rigdon (2001) and Knoth (2014?).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

L. A. Jones, C. W. Champ, S. E. Rigdon (2001), The performance of exponentially weighted moving average charts with estimated parameters, *Technometrics 43*, 156-167.

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2006), Computation of the ARL for CUSUM-S^2 schemes, *Computational Statistics & Data Analysis 51*, 499-512.

See Also

`sewma.arl` for zero-state ARL function of EWMA control charts w/o pre run uncertainty.

Examples

```
## Knoth (2014?)
```

sewma.crit *Compute critical values of EWMA control charts (variance charts)*

Description

Computation of the critical values (similar to alarm limits) for different types of EWMA control charts (based on the sample variance S^2) monitoring normal variance.

Usage

```
sewma.crit(l,L0,df,sigma0=1,cl=NULL,cu=NULL,hs=NULL,s2.on=TRUE,
sided="upper",mode="fixed",ur=4,r=40,qm=30)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`L0`	in-control ARL.
`df`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`sigma0`	in-control standard deviation.
`cl`	deployed for `sided="Rupper"`, that is, upper variance control chart with lower reflecting barrier `cl`.
`cu`	for two-sided (`sided="two"`) and fixed upper control limit (`mode="fixed"`) a value larger than `sigma0` has to been given, for all other cases `cu` is ignored.
`hs`	so-called headstart (enables fast initial response); the default (`NULL`) yields the expected in-control value of S^2 (1) and S (c_4), respectively.
`s2.on`	distinguishes between S^2 and S chart.
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`mode`	only deployed for `sided="two"` – with `"fixed"` an upper control limit (see `cu`) is set and only the lower is calculated to obtain the in-control ARL `L0`, while with `"unbiased"` a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated). With `"vanilla"` limits symmetric around 1 (the in-control value of the variance) are determined, while for `"eq.tails"` the in-control ARL values of two single EWMA variance charts (decompose the two-sided scheme into one lower and one upper scheme) are matched.
`ur`	truncation of lower chart for `eq.tails` mode.
`r`	dimension of the resulting linear equation system (highest order of the collocation polynomials).
`qm`	number of quadrature nodes for calculating the collocation definite integrals.

Details

sewma.crit determines the critical values (similar to alarm limits) for given in-control ARL L0 by applying secant rule and using sewma.arl(). In case of sided="two" and mode="unbiased" a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at sigma0. See Knoth (2010) and the related example.

Value

Returns the lower and upper control limit cl and cu.

Author(s)

Sven Knoth

References

H.-J. Mittag and D. Stemann and B. Tewes (1998), EWMA-Karten zur \"Uberwachung der Streuung von Qualit\"atsmerkmalen, *Allgemeines Statistisches Archiv 82*, 327-338,

C. A. Acosta-Mej\'ia and J. J. Pignatiello Jr. and B. V. Rao (1999), A comparison of control charting procedures for monitoring process dispersion, *IIE Transactions 31*, 569-579.

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2006a), Computation of the ARL for CUSUM-S^2 schemes, *Computational Statistics & Data Analysis 51*, 499-512.

S. Knoth (2006b), The art of evaluating monitoring schemes – how to measure the performance of control charts? in *Frontiers in Statistical Quality Control 8*, H.-J. Lenz and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 74-99.

S. Knoth (2010), Control Charting Normal Variance – Reflections, Curiosities, and Recommendations, in *Frontiers in Statistical Quality Control 9*, H.-J. Lenz and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 3-18.

See Also

sewma.arl for calculation of ARL of variance charts.

Examples

```
## Mittag et al. (1998)
## compare their upper critical value 2.91 that
## leads to the upper control limit via the formula shown below
## (for the usual upper EWMA \eqn{S^2}{S^2}).
## See Knoth (2006b) for a discussion of this EWMA setup and it's evaluation.

l  <- 0.18
L0 <- 250
df <- 4
limits <- sewma.crit(l, L0, df)
limits["cu"]
```

```
limits.cu.mittag_et_al <- 1 + sqrt(l/(2-l))*sqrt(2/df)*2.91
limits.cu.mittag_et_al

## Knoth (2005)
## reproduce the critical value given in Figure 2 (c=1.661865) for
## upper EWMA \eqn{S^2}{S^2} with df=1

l  <- 0.025
L0 <- 250
df <- 1
limits <- sewma.crit(l, L0, df)
cv.Fig2 <- (limits["cu"]-1)/( sqrt(l/(2-l))*sqrt(2/df) )
cv.Fig2

## the small difference (sixth digit after decimal point) stems from
## tighter criterion in the secant rule implemented in the R package.

## demo of unbiased ARL curves
## Deploy, please, not matrix dimensions smaller than 50 -- for the
## sake of accuracy, the value 80 was used.
## Additionally, this example needs between 1 and 2 minutes on a 1.6 Ghz box.

## Not run:
l  <- 0.1
L0 <- 500
df <- 4
limits <- sewma.crit(l, L0, df, sided="two", mode="unbiased", r=80)
SEWMA.arl <- Vectorize(sewma.arl, "sigma")
SEWMA.ARL <- function(sigma)
  SEWMA.arl(l, limits[1], limits[2], sigma, df, sided="two", r=80)
layout(matrix(1:2, nrow=1))
curve(SEWMA.ARL, .75, 1.25, log="y")
curve(SEWMA.ARL, .95, 1.05, log="y")
## End(Not run)
# the above stuff needs about 1 minute

## control limits for upper and lower EWMA charts with reflecting barriers
## (reflection at in-control level sigma0 = 1)
## examples from Knoth (2006a), Tables 4 and 5

## Not run:
## upper chart with reflection at sigma0=1 in Table 4: c = 2.4831
l <- 0.15
L0 <- 100
df <- 4
limits <- sewma.crit(l, L0, df, cl=1, sided="Rupper", r=100)
cv.Tab4 <- (limits["cu"]-1)/( sqrt(l/(2-l))*sqrt(2/df) )
cv.Tab4

## lower chart with reflection at sigma0=1 in Table 5: c = 2.0613
l <- 0.115
L0 <- 200
```

```
df <- 5
limits <- sewma.crit(l, L0, df, cu=1, sided="Rlower", r=100)
cv.Tab5 <- -(limits["cl"]-1)/( sqrt(l/(2-l))*sqrt(2/df) )
cv.Tab5
## End(Not run)
```

---

sewma.crit.prerun — *Compute critical values of of EWMA (variance charts) control charts under pre-run uncertainty*

Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal variance.

Usage

```
sewma.crit.prerun(l,L0,df1,df2,sigma0=1,cl=NULL,cu=NULL,hs=1,sided="upper",
mode="fixed",r=40,qm=30,qm.sigma=30,truncate=1e-10,
tail_approx=TRUE,c.error=1e-10,a.error=1e-9)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`L0`	in-control quantile value.
`df1`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`df2`	degrees of freedom of the pre-run variance estimator.
`sigma,sigma0`	true and in-control standard deviation, respectively.
`cl`	deployed for `sided="Rupper"`, that is, upper variance control chart with lower reflecting barrier `cl`.
`cu`	for two-sided (`sided="two"`) and fixed upper control limit (`mode="fixed"`) a value larger than `sigma0` has to been given, for all other cases `cu` is ignored.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`),and `"two"` (two-sided chart), respectively.
`mode`	only deployed for `sided="two"` – with `"fixed"` an upper control limit (see `cu`) is set and only the lower is calculated to obtain the in-control ARL `L0`, while with `"unbiased"` a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
`r`	dimension of the resulting linear equation system (highest order of the collocation polynomials).

qm	number of quadrature nodes for calculating the collocation definite integrals.
qm.sigma	number of quadrature nodes for convoluting the standard deviation uncertainty.
truncate	size of truncated tail.
tail_approx	controls whether the geometric tail approximation is used (is faster) or not.
c.error	error bound for two succeeding values of the critical value during applying the secant rule.
a.error	error bound for the quantile level alpha during applying the secant rule.

Details

sewma.crit.prerun determines the critical values (similar to alarm limits) for given in-control ARL L0 by applying secant rule and using sewma.arl.prerun(). In case of sided="two" and mode="unbiased" a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at sigma0. See Knoth (2010) for some details of the algorithm involved.

Value

Returns the lower and upper control limit cl and cu.

Author(s)

Sven Knoth

References

H.-J. Mittag and D. Stemann and B. Tewes (1998), EWMA-Karten zur \"Uberwachung der Streuung von Qualit\"atsmerkmalen, *Allgemeines Statistisches Archiv 82*, 327-338, S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2010), Control Charting Normal Variance – Reflections, Curiosities, and Recommendations, in *Frontiers in Statistical Quality Control 9*, H.-J. Lenz and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 3-18.

See Also

sewma.arl.prerun for calculation of ARL of variance charts under pre-run uncertainty and sewma.crit for the algorithm w/o pre-run uncertainty.

Examples

```
## Knoth (2014?)
```

`sewma.q` *Compute RL quantiles of EWMA (variance charts) control charts*

Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal variance.

Usage

```
sewma.q(l, cl, cu, sigma, df, alpha, hs=1, sided="upper", r=40, qm=30)

sewma.q.crit(l,L0,alpha,df,sigma0=1,cl=NULL,cu=NULL,hs=1,sided="upper",
mode="fixed",ur=4,r=40,qm=30,c.error=1e-12,a.error=1e-9)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`cl`	deployed for `sided="Rupper"`, that is, upper variance control chart with lower reflecting barrier `cl`.
`cu`	for two-sided (`sided="two"`) and fixed upper control limit (`mode="fixed"`) a value larger than `sigma0` has to been given, for all other cases `cu` is ignored.
`sigma,sigma0`	true and in-control standard deviation, respectively.
`df`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`alpha`	quantile level.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`),and `"two"` (two-sided chart), respectively.
`mode`	only deployed for `sided="two"` – with `"fixed"` an upper control limit (see `cu`) is set and only the lower is calculated to obtain the in-control ARL `L0`, while with `"unbiased"` a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
`ur`	truncation of lower chart for `classic` mode.
`r`	dimension of the resulting linear equation system (highest order of the collocation polynomials).
`qm`	number of quadrature nodes for calculating the collocation definite integrals.
`L0`	in-control quantile value.
`c.error`	error bound for two succeeding values of the critical value during applying the secant rule.
`a.error`	error bound for the quantile level `alpha` during applying the secant rule.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure. Thereby the ideas presented in Knoth (2007) are used. `sewma.q.crit` determines the critical values (similar to alarm limits) for given in-control RL quantile `L0` at level `alpha` by applying secant rule and using `sewma.sf()`. In case of `sided="two"` and `mode="unbiased"` a two-dimensional secant rule is applied that also ensures that the minimum of the cdf for given standard deviation is attained at `sigma0`.

Value

Returns a single value which resembles the RL quantile of order `alpha` and the lower and upper control limit `cl` and `cu`, respectively.

Author(s)

Sven Knoth

References

H.-J. Mittag and D. Stemann and B. Tewes (1998), EWMA-Karten zur \"Uberwachung der Streuung von Qualit\"atsmerkmalen, *Allgemeines Statistisches Archiv 82*, 327-338,

C. A. Acosta-Mej\'ia and J. J. Pignatiello Jr. and B. V. Rao (1999), A comparison of control charting procedures for monitoring process dispersion, *IIE Transactions 31*, 569-579.

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

S. Knoth (2010), Control Charting Normal Variance – Reflections, Curiosities, and Recommendations, in *Frontiers in Statistical Quality Control 9*, H.-J. Lenz and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 3-18.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`sewma.arl` for calculation of ARL of variance charts and `sewma.sf` for the RL survival function.

Examples

```
## Knoth (2014?)
```

sewma.q.prerun *Compute RL quantiles of EWMA (variance charts) control charts under pre-run uncertainty*

Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal variance.

Usage

```
sewma.q.prerun(l,cl,cu,sigma,df1,df2,alpha,hs=1,sided="upper",
r=40,qm=30,qm.sigma=30,truncate=1e-10)

sewma.q.crit.prerun(l,L0,alpha,df1,df2,sigma0=1,cl=NULL,cu=NULL,hs=1,
sided="upper",mode="fixed",r=40, qm=30,qm.sigma=30,truncate=1e-10,
tail_approx=TRUE,c.error=1e-10,a.error=1e-9)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`cl`	deployed for `sided="Rupper"`, that is, upper variance control chart with lower reflecting barrier `cl`.
`cu`	for two-sided (`sided="two"`) and fixed upper control limit (`mode="fixed"`) a value larger than `sigma0` has to been given, for all other cases `cu` is ignored.
`sigma,sigma0`	true and in-control standard deviation, respectively.
`L0`	in-control quantile value.
`alpha`	quantile level.
`df1`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`df2`	degrees of freedom of the pre-run variance estimator.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`mode`	only deployed for `sided="two"` – with `"fixed"` an upper control limit (see `cu`) is set and only the lower is calculated to obtain the in-control ARL `L0`, while with `"unbiased"` a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
`r`	dimension of the resulting linear equation system (highest order of the collocation polynomials).

qm	number of quadrature nodes for calculating the collocation definite integrals.
qm.sigma	number of quadrature nodes for convoluting the standard deviation uncertainty.
truncate	size of truncated tail.
tail_approx	controls whether the geometric tail approximation is used (is faster) or not.
c.error	error bound for two succeeding values of the critical value during applying the secant rule.
a.error	error bound for the quantile level alpha during applying the secant rule.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure. Thereby the ideas presented in Knoth (2007) are used. `sewma.q.crit.prerun` determines the critical values (similar to alarm limits) for given in-control RL quantile `L0` at level `alpha` by applying secant rule and using `sewma.sf()`. In case of `sided="two"` and `mode="unbiased"` a two-dimensional secant rule is applied that also ensures that the minimum of the cdf for given standard deviation is attained at `sigma0`.

Value

Returns a single value which resembles the RL quantile of order `alpha` and the lower and upper control limit `cl` and `cu`, respectively.

Author(s)

Sven Knoth

References

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`sewma.q` and `sewma.q.crit` for the version w/o pre-run uncertainty.

Examples

```
## Knoth (2014?)
```

`sewma.sf` *Compute the survival function of EWMA run length*

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal variance.

Usage

```
sewma.sf(n, l, cl, cu, sigma, df, hs=1, sided="upper", r=40, qm=30)
```

Arguments

`n`	calculate sf up to value `n`.
`l`	smoothing parameter lambda of the EWMA control chart.
`cl`	lower control limit of the EWMA control chart.
`cu`	upper control limit of the EWMA control chart.
`sigma`	true standard deviation.
`df`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`r`	dimension of the resulting linear equation system (highest order of the collocation polynomials).
`qm`	number of quadrature nodes for calculating the collocation definite integrals.

Details

The survival function P(L>n) and derived from it also the cdf P(L<=n) and the pmf P(L=n) illustrate the distribution of the EWMA run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann's survival function iteration procedure and on results in Knoth (2007).

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`sewma.arl` for zero-state ARL computation of variance EWMA control charts.

Examples

```
## Knoth (2014?)
```

`sewma.sf.prerun` *Compute the survival function of EWMA run length*

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal variance.

Usage

```
sewma.sf.prerun(n, l, cl, cu, sigma, df1, df2, hs=1, sided="upper",
qm=30, qm.sigma=30, truncate=1e-10, tail_approx=TRUE)
```

Arguments

`n`	calculate sf up to value `n`.
`l`	smoothing parameter lambda of the EWMA control chart.
`cl`	lower control limit of the EWMA control chart.
`cu`	upper control limit of the EWMA control chart.
`sigma`	true standard deviation.
`df1`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`df2`	degrees of freedom of the pre-run variance estimator.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`qm`	number of quadrature nodes for calculating the collocation definite integrals.

`qm.sigma`	number of quadrature nodes for convoluting the standard deviation uncertainty.
`truncate`	size of truncated tail.
`tail_approx`	Controls whether the geometric tail approximation is used (is faster) or not.

Details

The survival function P(L>n) and derived from it also the cdf P(L<=n) and the pmf P(L=n) illustrate the distribution of the EWMA run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann's survival function iteration procedure and on results in Knoth (2007)...

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`sewma.sf` for the RL survival function of EWMA control charts w/o pre-run uncertainty.

Examples

```
## Knoth (2014?)
```

`tol.lim.fac` *Two-sided tolerance limit factors*

Description

For constructing tolerance intervals, which cover a given proportion p of a normal distribution with unknown mean and variance with confidence $1 - \alpha$, one needs to calculate the so-called tolerance limit factors k. These values are computed for a given sample size n.

Usage

```
tol.lim.fac(n,p,a,mode="WW",m=30)
```

Arguments

`n`	sample size.
`p`	coverage.
`a`	error probability α, resulting interval covers at least proportion `p` with confidence of at least $1-\alpha$.
`mode`	distinguish between Wald/Wolfowitz' approximation method (`"WW"`) and the more accurate approach (`"exact"`) based on Gauss-Legendre quadrature.
`m`	number of abscissas for the quadrature (needed only for `method="exact"`), of course, the larger the more accurate.

Details

`tol.lim.fac` determines tolerance limits factors k by means of the fast and simple approximation due to Wald/Wolfowitz (1946) and of Gauss-Legendre quadrature like Odeh/Owen (1980), respectively, who used in fact the Simpson Rule. Then, by $\bar{x} \pm k \cdot s$ one can build the tolerance intervals which cover at least proportion p of a normal distribution for given confidence level of $1-\alpha$. $\bar{x}$ and s stand for the sample mean and the sample standard deviation, respectively.

Value

Returns a single value which resembles the tolerance limit factor.

Author(s)

Sven Knoth

References

A. Wald, J. Wolfowitz (1946), Tolerance limits for a normal distribution, *Annals of Mathematical Statistics 17*, 208-215.

R. E. Odeh, D. B. Owen (1980), *Tables for Normal Tolerance Limits*, Sampling Plans, and Screening, Marcel Dekker, New York.

See Also

`qnorm` for the ”asymptotic” case – cf. second example.

Examples

```
n <- 2:10
p <- .95
a <- .05
kWW <- sapply(n,p=p,a=a,tol.lim.fac)
kEX <- sapply(n,p=p,a=a,mode="exact",tol.lim.fac)
print(cbind(n,kWW,kEX),digits=4)
## Odeh/Owen (1980), page 98, in Table 3.4.1
##  n  factor k
##  2  36.519
```

```
## 3   9.789
## 4   6.341
## 5   5.077
## 6   4.422
## 7   4.020
## 8   3.746
## 9   3.546
## 10  3.393

## n -> infty
n <- 10^{1:7}
p <- .95
a <- .05
kEX <- round(sapply(n,p=p,a=a,mode="exact",tol.lim.fac),digits=4)
kEXinf <- round(qnorm(1-a/2),digits=4)
print(rbind(cbind(n,kEX),c("infinity",kEXinf)),quote=FALSE)
```

`x.res.ewma.arl` *Compute ARLs of EWMA residual control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) for EWMA residual control charts monitoring normal mean, variance, or mean and variance simultaneously. Additionally, the probability of misleading signals (PMS) is calculated.

Usage

```
x.res.ewma.arl(l, c, mu, alpha=0, n=5, hs=0, r=40)

s.res.ewma.arl(l, cu, sigma, mu=0, alpha=0, n=5, hs=1, r=40, qm=30)

xs.res.ewma.arl(lx, cx, ls, csu, mu, sigma, alpha=0,
n=5, hsx=0, rx=40, hss=1, rs=40, qm=30)

xs.res.ewma.pms(lx, cx, ls, csu, mu, sigma, type="3",
alpha=0, n=5, hsx=0, rx=40, hss=1, rs=40, qm=30)
```

Arguments

`l, lx, ls`	smoothing parameter(s) lambda of the EWMA control chart.
`c, cu, cx, csu`	critical value (similar to alarm limit) of the EWMA control charts.
`mu`	true mean.
`sigma`	true standard deviation.
`alpha`	the AR(1) coefficient – first order autocorrelation of the original data.
`n`	batch size.
`hs, hsx, hss`	so-called headstart (enables fast initial response).

`r, rx, rs`	number of quadrature nodes or size of collocation base, dimension of the resulting linear equation system is equal to `r` (two-sided).
`qm`	number of nodes for collocation quadratures.
`type`	PMS type, for `PMS="3"` (the default) the probability of getting a mean signal despite the variance changed, and for `PMS="4"` the opposite case is dealt with.

Details

The above list of functions provides the application of algorithms developed for iid data to the residual case. To be more precise, the underlying model is a sequence of normally distributed batches with size `n` with autocorrelation within the batch and independence between the batches (see also the references below). It is restricted to the classical EWMA chart types, that is two-sided for the mean, upper charts for the variance, and all equipped with fixed limits. The autocorrelation is modeled by an AR(1) process with parameter `alpha`. Additionally, with `xs.res.ewma.pms` the probability of misleading signals (PMS) of `type` is calculated. This is offered exclusively in this small collection so that for iid data this function has to be used too (with `alpha=0`).

Value

Return single values which resemble the ARL and the PMS, respectively.

Author(s)

Sven Knoth

References

S. Knoth, M. C. Morais, A. Pacheco, W. Schmid (2009), Misleading Signals in Simultaneous Residual Schemes for the Mean and Variance of a Stationary Process, *Commun. Stat., Theory Methods 38*, 2923-2943.

S. Knoth, W. Schmid, A. Schoene (2001), Simultaneous Shewhart-Type Charts for the Mean and the Variance of a Time Series, *Frontiers of Statistical Quality Control 6, A. Lenz, H.-J. & Wilrich, P.-T. (Eds.)*, 6, 61-79.

S. Knoth, W. Schmid (2002) Monitoring the mean and the variance of a stationary process, *Statistica Neerlandica 56*, 77-100.

See Also

`xewma.arl`, `sewma.arl`, and `xsewma.arl` as more elaborated functions in the iid case.

Examples

```
## Not run:
## S. Knoth, W. Schmid (2002)

cat("\nFragments of Table 2 (n=5, lambda.1=lambda.2)\n")

lambdas <- c(.5, .25, .1, .05)
L0 <- 500
```

```
n <- 5

crit <- NULL
for ( lambda in lambdas ) {
  cs <- xsewma.crit(lambda, lambda, L0, n-1)
  x.e <- round(cs[1], digits=4)
  names(x.e) <- NULL
  s.e <- round((cs[3]-1) * sqrt((2-lambda)/lambda)*sqrt((n-1)/2), digits=4)
  names(s.e) <- NULL
  crit <- rbind(crit, data.frame(lambda, x.e, s.e))
}


## orinal values are (Markov chain approximation with 50 states)
# lambda x.e    s.e
#   0.50 3.2765 4.6439
#   0.25 3.2168 4.0149
#   0.10 3.0578 3.3376
#   0.05 2.8817 2.9103

print(crit)


cat("\nFragments of Table 4 (n=5, lambda.1=lambda.2=0.1)\n\n")

lambda <- .1
# the algorithm used in Knoth/Schmid is less accurate -- proceed with their values
cx <- x.e <- 3.0578
s.e <- 3.3376
csu <- 1 + s.e * sqrt(lambda/(2-lambda))*sqrt(2/(n-1))

alpha <- .3

a.values <- c((0:6)/4, 2)
d.values <- c(1 + (0:5)/10, 1.75 , 2)

arls <- NULL
for ( delta in d.values ) {
  row <- NULL
  for ( mu in a.values ) {
   arl <- round(xs.res.ewma.arl(lambda, cx, lambda, csu, mu*sqrt(n), delta, alpha=alpha, n=n),
                 digits=2)
    names(arl) <- NULL
    row <- c(row, arl)
  }
  arls <- rbind(arls, data.frame(t(row)))
}
names(arls) <- a.values
rownames(arls) <- d.values

## orinal values are (now Monte-Carlo with 10^6 replicates)
#          0  0.25   0.5 0.75    1 1.25  1.5    2
#1    502.44 49.50 14.21 7.93 5.53 4.28 3.53 2.65
```

```
#1.1   73.19 32.91 13.33 7.82 5.52 4.29 3.54 2.66
#1.2   24.42 18.88 11.37 7.44 5.42 4.27 3.54 2.67
#1.3   13.11 11.83  9.09 6.74 5.18 4.17 3.50 2.66
#1.4    8.74  8.31  7.19 5.89 4.81 4.00 3.41 2.64
#1.5    6.50  6.31  5.80 5.08 4.37 3.76 3.28 2.59
#1.75   3.94  3.90  3.78 3.59 3.35 3.09 2.83 2.40
#2      2.85  2.84  2.80 2.73 2.63 2.51 2.39 2.14

print(arls)


## S. Knoth, M. C. Morais, A. Pacheco, W. Schmid (2009)

cat("\nFragments of Table 5 (n=5, lambda=0.1)\n\n")

d.values <- c(1.02, 1 + (1:5)/10, 1.75 , 2)

arl.x <- arl.s <- arl.xs <- PMS.3 <- NULL
for ( delta in d.values ) {
  arl.x  <- c(arl.x,  round(x.res.ewma.arl(lambda, cx/delta, 0, n=n),
                            digits=3))
  arl.s  <- c(arl.s,  round(s.res.ewma.arl(lambda, csu, delta, n=n),
                            digits=3))
  arl.xs <- c(arl.xs, round(xs.res.ewma.arl(lambda, cx, lambda, csu, 0, delta, n=n),
                            digits=3))
  PMS.3  <- c(PMS.3,  round(xs.res.ewma.pms(lambda, cx, lambda, csu, 0, delta, n=n),
                            digits=6))
}

## orinal values are (Markov chain approximation)
# delta   arl.x   arl.s  arl.xs PMS.3
#  1.02 833.086 518.935 323.324 0.381118
#  1.10 454.101  84.208  73.029 0.145005
#  1.20 250.665  25.871  24.432 0.071024
#  1.30 157.343  13.567  13.125 0.047193
#  1.40 108.112   8.941   8.734 0.035945
#  1.50  79.308   6.614   6.493 0.029499
#  1.75  44.128   3.995   3.942 0.021579
#  2.00  28.974   2.887   2.853 0.018220

print(cbind(delta=d.values, arl.x, arl.s, arl.xs, PMS.3))


cat("\nFragments of Table 6 (n=5, lambda=0.1)\n\n")

alphas <- c(-0.9, -0.5, -0.3, 0, 0.3, 0.5, 0.9)
deltas <- c(0.05, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 2)

PMS.4 <- NULL
for ( ir in 1:length(deltas) ) {
  mu <- deltas[ir]*sqrt(n)
  pms <- NULL
  for ( alpha in alphas ) {
```

```
    pms <- c(pms, round(xs.res.ewma.pms(lambda, cx, lambda, csu, mu, 1, type="4", alpha=alpha, n=n),
                        digits=6))
  }
  PMS.4 <- rbind(PMS.4, data.frame(delta=deltas[ir], t(pms)))
}
names(PMS.4) <- c("delta", alphas)
rownames(PMS.4) <- NULL

## orinal values are (Markov chain approximation)
#  delta     -0.9     -0.5     -0.3        0      0.3      0.5      0.9
#   0.05 0.055789 0.224521 0.279842 0.342805 0.391299 0.418915 0.471386
#   0.25 0.003566 0.009522 0.014580 0.025786 0.044892 0.066584 0.192023
#   0.50 0.002994 0.001816 0.002596 0.004774 0.009259 0.015303 0.072945
#   0.75 0.006967 0.000703 0.000837 0.001529 0.003400 0.006424 0.046602
#   1.00 0.005098 0.000402 0.000370 0.000625 0.001589 0.003490 0.039978
#   1.25 0.000084 0.000266 0.000202 0.000300 0.000867 0.002220 0.039773
#   1.50 0.000000 0.000256 0.000120 0.000163 0.000531 0.001584 0.042734
#   2.00 0.000000 0.000311 0.000091 0.000056 0.000259 0.001029 0.054543

print(PMS.4)

## End(Not run)
```

`xcusum.ad` *Compute steady-state ARLs of CUSUM control charts*

Description

Computation of the steady-state Average Run Length (ARL) for different types of CUSUM control charts monitoring normal mean.

Usage

```
xcusum.ad(k, h, mu1, mu0 = 0, sided = "one", r = 30)
```

Arguments

`k`	reference value of the CUSUM control chart.
`h`	decision interval (alarm limit, threshold) of the CUSUM control chart.
`mu1`	out-of-control mean.
`mu0`	in-control mean.
`sided`	distinguish between one-, two-sided and Crosier's modified two-sided CUSUM scheme by choosing `"one"`, `"two"`, and `"Crosier"`, respectively.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-, two-sided) or `2r+1` (Crosier).

Details

`xcusum.ad` determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature and using the power method for deriving the largest in magnitude eigenvalue and the related left eigenfunction.

Value

Returns a single value which resembles the steady-state ARL.

Note

Be cautious in increasing the dimension parameter `r` for two-sided CUSUM schemes. The resulting matrix dimension is `r^2` times `r^2`. Thus, go beyond 30 only on fast machines. This is the only case, were the package routines are based on the Markov chain approach. Moreover, the two-sided CUSUM scheme needs a two-dimensional Markov chain.

Author(s)

Sven Knoth

References

R. B. Crosier (1986), A new two-sided cumulative quality control scheme, *Technometrics 28*, 187-194.

See Also

`xcusum.arl` for zero-state ARL computation and `xewma.ad` for the steady-state ARL of EWMA control charts.

Examples

```
## comparison of zero-state (= worst case ) and steady-state performance
## for one-sided CUSUM control charts

k <- .5
h <- xcusum.crit(k,500)
mu <- c(0,.5,1,1.5,2)
arl <- sapply(mu,k=k,h=h,xcusum.arl)
ad <- sapply(mu,k=k,h=h,xcusum.ad)
round(cbind(mu,arl,ad),digits=2)

## Crosier (1986), Crosier's modified two-sided CUSUM
## He introduced the modification and evaluated it by means of
## Markov chain approximation

k <- .5
h2 <- 4
hC <- 3.73
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,4,5)
```

```
ad2 <- sapply(mu,k=k,h=h2,sided="two",r=20,xcusum.ad)
adC <- sapply(mu,k=k,h=hC,sided="Crosier",xcusum.ad)
round(cbind(mu,ad2,adC),digits=2)

## results in the original paper are (in Table 5)
## 0.00 163.   164.
## 0.25  71.6   69.0
## 0.50  25.2   24.3
## 0.75  12.3   12.1
## 1.00   7.68   7.69
## 1.50   4.31   4.39
## 2.00   3.03   3.12
## 2.50   2.38   2.46
## 3.00   2.00   2.07
## 4.00   1.55   1.60
## 5.00   1.22   1.29
```

`xcusum.arl` *Compute ARLs of CUSUM control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM control charts monitoring normal mean.

Usage

```
xcusum.arl(k, h, mu, hs = 0, sided = "one", method = "igl", q = 1, r = 30)
```

Arguments

`k`	reference value of the CUSUM control chart.
`h`	decision interval (alarm limit, threshold) of the CUSUM control chart.
`mu`	true mean.
`hs`	so-called headstart (give fast initial response).
`sided`	distinguish between one-, two-sided and Crosier's modified two-sided CUSUM scheme by choosing `"one"`, `"two"`, and `"Crosier"`, respectively.
`method`	deploy the integral equation (`"igl"`) or Markov chain approximation (`"mc"`) method to calculate the ARL (currently only for two-sided CUSUM implemented).
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu != 0$ conditional delays, that is, $E_q(L - q + 1 \mid L \geq q)$, will be determined. Note that mu0=0 is implicitely fixed.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-, two-sided) or `2r+1` (Crosier).

Details

xcusum.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

Value

Returns a vector of length q which resembles the ARL and the sequence of conditional expected delays for q=1 and q>1, respectively.

Author(s)

Sven Knoth

References

A. L. Goel, S. M. Wu (1971), Determination of A.R.L. and a contour nomogram for CUSUM charts to control normal mean, *Technometrics 13*, 221-230.

D. Brook, D. A. Evans (1972), An approach to the probability distribution of cusum run length, *Biometrika 59*, 539-548.

J. M. Lucas, R. B. Crosier (1982), Fast initial response for cusum quality-control schemes: Give your cusum a headstart, *Technometrics 24*, 199-205.

L. C. Vance (1986), Average run lengths of cumulative sum control charts for controlling normal means, *Journal of Quality Technology 18*, 189-193.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of one-sided and two-sided CUSUM quality control schemes, *Technometrics 28*, 61-67.

R. B. Crosier (1986), A new two-sided cumulative quality control scheme, *Technometrics 28*, 187-194.

See Also

xewma.arl for zero-state ARL computation of EWMA control charts and xcusum.ad for the steady-state ARL.

Examples

```
## Brook/Evans (1972), one-sided CUSUM
## Their results are based on the less accurate Markov chain approach.

k <- .5
h <- 3
round(c( xcusum.arl(k,h,0), xcusum.arl(k,h,1.5) ),digits=2)

## results in the original paper are L0 = 117.59, L1 = 3.75 (in Subsection 4.3).

## Lucas, Crosier (1982)
## (one- and) two-sided CUSUM with possible headstarts

k <- .5
h <- 4
```

```
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,4,5)
arl1 <- sapply(mu,k=k,h=h,sided="two",xcusum.arl)
arl2 <- sapply(mu,k=k,h=h,hs=h/2,sided="two",xcusum.arl)
round(cbind(mu,arl1,arl2),digits=2)

## results in the original paper are (in Table 1)
## 0.00 168.   149.
## 0.25  74.2   62.7
## 0.50  26.6   20.1
## 0.75  13.3    8.97
## 1.00   8.38   5.29
## 1.50   4.75   2.86
## 2.00   3.34   2.01
## 2.50   2.62   1.59
## 3.00   2.19   1.32
## 4.00   1.71   1.07
## 5.00   1.31   1.01

## Vance (1986), one-sided CUSUM
## The first paper on using Nystroem method and Gauss-Legendre quadrature
## for solving the ARL integral equation (see as well Goel/Wu, 1971)

k <- 0
h <- 10
mu <- c(-.25,-.125,0,.125,.25,.5,.75,1)
round(cbind(mu,sapply(mu,k=k,h=h,xcusum.arl)),digits=2)

## results in the original paper are (in Table 1 incl. Goel/Wu (1971) results)
##  -0.25  2071.51
##  -0.125  400.28
##   0.0    124.66
##   0.125   59.30
##   0.25    36.71
##   0.50    20.37
##   0.75    14.06
##   1.00    10.75

## Waldmann (1986),
## one- and two-sided CUSUM

## one-sided case

k <- .5
h <- 3
mu <- c(-.5,0,.5)
round(sapply(mu,k=k,h=h,xcusum.arl),digits=2)

## results in the original paper are 1963, 117.4, and 17.35, resp.
## (in Tables 3, 1, and 5, resp.).

## two-sided case

k <- .6
```

```
h <- 3
round(xcusum.arl(k,h,-.2,sided="two"),digits=1)  # fits to Waldmann's setup

## result in the original paper is 65.4 (in Table 6).

## Crosier (1986), Crosier's modified two-sided CUSUM
## He introduced the modification and evaluated it by means of
## Markov chain approximation

k <- .5
h <- 3.73
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,4,5)
round(cbind(mu,sapply(mu,k=k,h=h,sided="Crosier",xcusum.arl)),digits=2)

## results in the original paper are (in Table 3)
## 0.00 168.
## 0.25  70.7
## 0.50  25.1
## 0.75  12.5
## 1.00   7.92
## 1.50   4.49
## 2.00   3.17
## 2.50   2.49
## 3.00   2.09
## 4.00   1.60
## 5.00   1.22

## SAS/QC manual 1999
## one- and two-sided CUSUM schemes

## one-sided

k <- .25
h <- 8
mu <- 2.5
print(xcusum.arl(k,h,mu),digits=12)
print(xcusum.arl(k,h,mu,hs=.1),digits=12)

## original results are 4.1500836225 and 4.1061588131.

## two-sided

print(xcusum.arl(k,h,mu,sided="two"),digits=12)

## original result is 4.1500826715.
```

`xcusum.crit` *Compute decision intervals of CUSUM control charts*

Description

Computation of the decision intervals (alarm limits) for different types of CUSUM control charts monitoring normal mean.

Usage

```
xcusum.crit(k, L0, mu0 = 0, hs = 0, sided = "one", r = 30)
```

Arguments

k	reference value of the CUSUM control chart.
L0	in-control ARL.
mu0	in-control mean.
hs	so-called headstart (enables fast initial response).
sided	distinguishes between one-, two-sided and Crosier's modified two-sided CUSUM scheme by choosing "one", "two", and "Crosier", respectively.
r	number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-, two-sided) or 2r+1 (Crosier).

Details

xcusum.crit determines the decision interval (alarm limit) for given in-control ARL L0 by applying secant rule and using xcusum.arl().

Value

Returns a single value which resembles the decision interval h.

Author(s)

Sven Knoth

See Also

xcusum.arl for zero-state ARL computation.

Examples

```
k <- .5
incontrolARL <- c(500,5000,50000)
sapply(incontrolARL,k=k,xcusum.crit,r=10) # accuracy with 10 nodes
sapply(incontrolARL,k=k,xcusum.crit,r=20) # accuracy with 20 nodes
sapply(incontrolARL,k=k,xcusum.crit)      # accuracy with 30 nodes
```

xcusum.crit.L0h *Compute the CUSUM reference value k for given in-control ARL and threshold h*

Description

Computation of the reference value k for one-sided CUSUM control charts monitoring normal mean, if the in-control ARL L0 and the alarm threshold h are given.

Usage

```
xcusum.crit.L0h(L0, h, hs=0, sided="one", r=30, L0.eps=1e-6, k.eps=1e-8)
```

Arguments

L0	in-control ARL.
h	alarm level of the CUSUM control chart.
hs	so-called headstart (enables fast initial response).
sided	distinguishes between one-, two-sided and Crosier's modified two-sided CUSUM scheme choosing "one", "two", and "Crosier", respectively.
r	number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-, two-sided) or 2r+1 (Crosier).
L0.eps	error bound for the L0 error.
k.eps	bound for the difference of two successive values of k.

Details

xcusum.crit.L0h determines the reference value k for given in-control ARL L0 and alarm level h by applying secant rule and using xcusum.arl(). Note that not for any combination of L0 and h a solution exists – for given L0 there is a maximal value for h to get a valid result k.

Value

Returns a single value which resembles the reference value k.

Author(s)

Sven Knoth

See Also

xcusum.arl for zero-state ARL computation.

Examples

```
L0 <- 100
h.max <- xcusum.crit(0, L0, 0)
hs <- (300:1)/100
hs <- hs[hs < h.max]
ks <- NULL
for ( h in hs ) ks <- c(ks, xcusum.crit.L0h(L0, h))
k.max <- qnorm( 1 - 1/L0 )
plot(hs, ks, type="l", ylim=c(0, max(k.max, ks)), xlab="h", ylab="k")
abline(h=c(0, k.max), col="red")
```

`xcusum.crit.L0L1` *Compute the CUSUM k and h for given in-control ARL L0 and out-of-control L1*

Description

Computation of the reference value k and the alarm threshold h for one-sided CUSUM control charts monitoring normal mean, if the in-control ARL L0 and the out-of-control L1 are given.

Usage

```
xcusum.crit.L0L1(L0, L1, hs=0, sided="one", r=30, L1.eps=1e-6, k.eps=1e-8)
```

Arguments

`L0`	in-control ARL.
`L1`	out-of-control ARL.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one-, two-sided and Crosier's modified two-sided CUSUM schemoosing `"one"`, `"two"`, and `"Crosier"`, respectively.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-, two-sided) or `2r+1` (Crosier).
`L1.eps`	error bound for the L1 error.
`k.eps`	bound for the difference of two successive values of k.

Details

`xcusum.crit.L0L1` determines the reference value k and the alarm threshold h for given in-control ARL `L0` and out-of-control ARL `L1` by applying secant rule and using `xcusum.arl()` and `xcusum.crit()`. These CUSUM design rules were firstly (and quite rarely afterwards) used by Ewan and Kemp.

Value

Returns two values which resemble the reference value `k` and the threshold `h`.

Author(s)

Sven Knoth

References

W. D. Ewan and K. W. Kemp (1960), Sampling inspection of continuous processes with no autocorrelation between successive results, *Biometrika 47*, 363-380.

K. W. Kemp (1962), The Use of Cumulative Sums for Sampling Inspection Schemes, *Journal of the Royal Statistical Sociecty C, Applied Statistics, 10*, 16-31.

See Also

`xcusum.arl` for zero-state ARL and `xcusum.crit` for threshold h computation.

Examples

```
## Table 2 from Ewan/Kemp (1960) -- one-sided CUSUM
#
# A.R.L. at A.Q.L.   A.R.L. at A.Q.L.     k      h
#      1000                3          1.12   2.40
#      1000                7          0.65   4.06
#       500                3          1.04   2.26
#       500                7          0.60   3.80
#       250                3          0.94   2.11
#       250                7          0.54   3.51
#
L0.set <- c(1000, 500, 250)
L1.set <- c(3, 7)
cat("\nL0\tL1\tk\th\n")
for ( L0 in L0.set ) {
  for ( L1 in L1.set ) {
    result <- round(xcusum.crit.L0L1(L0, L1), digits=2)
    cat(paste(L0, L1, result[1], result[2], sep="\t"), "\n")
  }
}
#
# two confirmation runs
xcusum.arl(0.54, 3.51, 0) # Ewan/Kemp
xcusum.arl(result[1], result[2], 0) # here
xcusum.arl(0.54, 3.51, 2*0.54) # Ewan/Kemp
xcusum.arl(result[1], result[2], 2*result[1]) # here
#
## Table II from Kemp (1962) -- two-sided CUSUM
#
#    Lr                  k
#             La=250   La=500   La=1000
#    2.5       1.05     1.17     1.27
#    3.0       0.94     1.035    1.13
#    4.0       0.78     0.85     0.92
#    5.0       0.68     0.74     0.80
#    6.0       0.60     0.655    0.71
```

```
#    7.5       0.52     0.57     0.62
#   10.0       0.43     0.48     0.52
#
L0.set <- c(250, 500, 1000)
L1.set <- c(2.5, 3:6, 7.5, 10)
cat("\nL1\tL0=250\tL0=500\tL0=1000\n")
for ( L1 in L1.set ) {
  cat(L1)
  for ( L0 in L0.set ) {
    result <- round(xcusum.crit.L0L1(L0, L1, sided="two"), digits=2)
    cat("\t", result[1])
  }
  cat("\n")
}
```

`xcusum.q` *Compute RL quantiles of CUSUM control charts*

Description

Computation of quantiles of the Run Length (RL)for CUSUM control charts monitoring normal mean.

Usage

```
xcusum.q(k, h, mu, alpha, hs=0, sided="one", r=40)
```

Arguments

`k`	reference value of the CUSUM control chart.
`h`	decision interval (alarm limit, threshold) of the CUSUM control chart.
`mu`	true mean.
`alpha`	quantile level.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided CUSUM control chart by choosing `"one"` and `"two"`, respectively.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1`.

Details

Instead of the popular ARL (Average Run Length) quantiles of the CUSUM stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure.

Value

Returns a single value which resembles the RL quantile of order q.

Author(s)

Sven Knoth

References

K.-H. Waldmann (1986), Bounds for the distribution of the run length of one-sided and two-sided CUSUM quality control schemes, *Technometrics 28*, 61-67.

See Also

`xcusum.arl` for zero-state ARL computation of CUSUM control charts.

Examples

```
## Waldmann (1986), one-sided CUSUM, Table 2
## original values are 345, 82, 9

XCUSUM.Q <- Vectorize("xcusum.q", "alpha")
k <- .5
h <- 3
mu <- 0 # corresponds to Waldmann's -0.5
a.list <- c(.95, .5, .05)
rl.quantiles <- ceiling(XCUSUM.Q(k, h, mu, a.list))
cbind(a.list, rl.quantiles)
```

`xcusum.sf` *Compute the survival function of CUSUM run length*

Description

Computation of the survival function of the Run Length (RL) for CUSUM control charts monitoring normal mean.

Usage

```
xcusum.sf(k, h, mu, n, hs=0, sided="one", r=40)
```

Arguments

`k`	reference value of the CUSUM control chart.
`h`	decision interval (alarm limit, threshold) of the CUSUM control chart.
`mu`	true mean.
`n`	calculate sf up to value `n`.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided CUSUM control chart by choosing `"one"` and `"two"`, respectively.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1`.

Details

The survival function P(L>n) and derived from it also the cdf P(L<=n) and the pmf P(L=n) illustrate the distribution of the CUSUM run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann's survival function iteration procedure.

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

K.-H. Waldmann (1986), Bounds for the distribution of the run length of one-sided and two-sided CUSUM quality control schemes, *Technometrics 28*, 61-67.

See Also

`xcusum.q` for computation of CUSUM run length quantiles.

Examples

```
## Waldmann (1986), one-sided CUSUM, Table 2

k <- .5
h <- 3
mu <- 0 # corresponds to Waldmann's -0.5
SF <- xcusum.sf(k, h, 0, 1000)
plot(1:length(SF), SF, type="l", xlab="n", ylab="P(L>n)", ylim=c(0,1))
#
```

`xDcusum.arl` *Compute ARLs of CUSUM control charts under drift*

Description

Computation of the (zero-state and other) Average Run Length (ARL) under drift for one-sided CUSUM control charts monitoring normal mean.

Usage

```
xDcusum.arl(k, h, delta, hs = 0, sided = "one",
    mode = "Gan", m = NULL, q = 1, r = 30, with0 = FALSE)
```

Arguments

- `k` reference value of the CUSUM control chart.
- `h` decision interval (alarm limit, threshold) of the CUSUM control chart.
- `delta` true drift parameter.
- `hs` so-called headstart (enables fast initial response).
- `sided` distinguishes between one- and two-sided CUSUM control chart by choosing `"one"` and `"two"`, respectively. Currentlly, the two-sided scheme is not implemented.
- `mode` decide whether Gan's or Knoth's approach is used. Use `"Gan"` and `"Knoth"`, respectively.
- `m` parameter used if `mode="Gan"`. `m` is design parameter of Gan's approach. If `m=NULL`, then `m` will increased until the resulting ARL does not change anymore.
- `q` change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu! = 0$ conditional delays, that is, $E_q(L - q + 1|L \geq)$, will be determined. Note that mu0=0 is implicitely fixed. Deploy large `q` to mimic steady-state. It works only for `mode="Knoth"`.
- `r` number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).
- `with0` defines whether the first observation used for the RL calculation follows already 1*delta or still 0*delta. With `q` additional flexibility is given.

Details

Based on Gan (1991) or Knoth (2003), the ARL is calculated for one-sided CUSUM control charts under drift. In case of Gan's framework, the usual ARL function with mu=m*delta is determined and recursively via m-1, m-2, ... 1 (or 0) the drift ARL determined. The framework of Knoth allows to calculate ARLs for varying parameters, such as control limits and distributional parameters. For details see the cited papers. Note that two-sided CUSUM charts under drift are difficult to treat.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

F. F. Gan (1992), CUSUM control charts under linear drift, *Statistician 41*, 71-84.

F. F. Gan (1996), Average Run Lengths for Cumulative Sum control chart under linear trend, *Applied Statistics 45*, 505-512.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2012), More on Control Charting under Drift, in: *Frontiers in Statistical Quality Control 10*, H.-J. Lenz, W. Schmid and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 53-68.

C. Zou, Y. Liu and Z. Wang (2009), Comparisons of control schemes for monitoring the means of processes subject to drifts, *Metrika 70*, 141-163.

See Also

`xcusum.arl` and `xcusum.ad` for zero-state and steady-state ARL computation of CUSUM control charts for the classical step change model.

Examples

```
## Gan (1992)
## Table 1
## original values are
#   deltas  arl
#   0.0001  475
#   0.0005  261
#   0.0010  187
#   0.0020  129
#   0.0050  76.3
#   0.0100  52.0
#   0.0200  35.2
#   0.0500  21.4
#   0.1000  15.0
#   0.5000  6.95
#   1.0000  5.16
#   3.0000  3.30
## Not run: k <- .25
h <- 8
r <- 50
DxDcusum.arl <- Vectorize(xDcusum.arl, "delta")
deltas <- c(0.0001, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.5, 1, 3)
arl.like.Gan   <-
  round(DxDcusum.arl(k, h, deltas, r=r, with0=TRUE), digits=2)
arl.like.Knoth <-
  round(DxDcusum.arl(k, h, deltas, r=r, mode="Knoth", with0=TRUE), digits=2)
data.frame(deltas, arl.like.Gan, arl.like.Knoth)
## End(Not run)

## Zou et al. (2009)
## Table 1
## original values are
#  delta   arl1  arl2  arl3
#  0           ~ 1730
#  0.0005  345   412   470
#  0.001   231   275   317
#  0.005   86.6  98.6  112
#  0.01    56.9  61.8  69.3
#  0.05    22.6  21.6  22.7
#  0.1     15.4  14.7  14.2
#  0.5     6.60  5.54  5.17
```

```
#  1.0     4.63  3.80  3.45
#  2.0     3.17  2.67  2.32
#  3.0     2.79  2.04  1.96
#  4.0     2.10  1.98  1.74
## Not run:
k1 <- 0.25
k2 <- 0.5
k3 <- 0.75
h1 <- 9.660
h2 <- 5.620
h3 <- 3.904
deltas <- c(0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1:4)
arl1 <- c(round(xcusum.arl(k1, h1, 0, r=r), digits=2),
          round(DxDcusum.arl(k1, h1, deltas, r=r), digits=2))
arl2 <- c(round(xcusum.arl(k2, h2, 0), digits=2),
          round(DxDcusum.arl(k2, h2, deltas, r=r), digits=2))
arl3 <- c(round(xcusum.arl(k3, h3, 0, r=r), digits=2),
          round(DxDcusum.arl(k3, h3, deltas, r=r), digits=2))
data.frame(delta=c(0, deltas), arl1, arl2, arl3)
## End(Not run)
```

`xDewma.arl` *Compute ARLs of EWMA control charts under drift*

Description

Computation of the (zero-state and other) Average Run Length (ARL) under drift for different types of EWMA control charts monitoring normal mean.

Usage

```
xDewma.arl(l, c, delta, zr = 0, hs = 0, sided = "one", limits = "fix",
    mode = "Gan", m = NULL, q = 1, r = 40, with0 = FALSE)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`delta`	true drift parameter.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguish between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`mode`	decide whether Gan's or Knoth's approach is used. Use `"Gan"` and `"Knoth"`, respectively.

`m`	parameter used if `mode="Gan"`. `m` is design parameter of Gan's approach. If `m=NULL`, then `m` will increased until the resulting ARL does not change anymore.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu! = 0$ conditional delays, that is, $E_q(L - q + 1 \| L \geq)$, will be determined. Note that mu0=0 is implicitely fixed. Deploy large `q` to mimic steady-state. It works only for `mode="Knoth"`.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).
`with0`	defines whether the first observation used for the RL calculation follows already 1*delta or still 0*delta. With `q` additional flexibility is given.

Details

Based on Gan (1991) or Knoth (2003), the ARL is calculated for EWMA control charts under drift. In case of Gan's framework, the usual ARL function with mu=m*delta is determined and recursively via m-1, m-2, ... 1 (or 0) the drift ARL determined. The framework of Knoth allows to calculate ARLs for varying parameters, such as control limits and distributional parameters. For details see the cited papers.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

F. F. Gan (1991), EWMA control chart under linear drift, *J. Stat. Comput. Simulation 38*, 181-200.

L. A. Aerne, C. W. Champ and S. E. Rigdon (1991), Evaluation of control charts under linear trend, *Commun. Stat., Theory Methods 20*, 3341-3349.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

H. M. Fahmy and E. A. Elsayed (2006), Detection of linear trends in process mean, *International Journal of Production Research 44*, 487-504.

S. Knoth (2012), More on Control Charting under Drift, in: *Frontiers in Statistical Quality Control 10*, H.-J. Lenz, W. Schmid and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 53-68.

C. Zou, Y. Liu and Z. Wang (2009), Comparisons of control schemes for monitoring the means of processes subject to drifts, *Metrika 70*, 141-163.

See Also

`xewma.arl` and `xewma.ad` for zero-state and steady-state ARL computation of EWMA control charts for the classical step change model.

Examples

```
## Not run:
DxDewma.arl <- Vectorize(xDewma.arl, "delta")
## Gan (1991)
## Table 1
## original values are
#  delta   arlE1  arlE2  arlE3
#  0       500    500    500
#  0.0001  482    460    424
#  0.0010  289    231    185
#  0.0020  210    162    129
#  0.0050  126     94.6   77.9
#  0.0100   81.7   61.3   52.7
#  0.0500   27.5   21.8   21.9
#  0.1000   17.0   14.2   15.3
#  1.0000    4.09   4.28   5.25
#  3.0000    2.60   2.90   3.43
#
lambda1 <- 0.676
lambda2 <- 0.242
lambda3 <- 0.047
h1 <- 2.204/sqrt(lambda1/(2-lambda1))
h2 <- 1.111/sqrt(lambda2/(2-lambda2))
h3 <- 0.403/sqrt(lambda3/(2-lambda3))
deltas <- c(.0001, .001, .002, .005, .01, .05, .1, 1, 3)
arlE10 <- round(xewma.arl(lambda1, h1, 0, sided="two"), digits=2)
arlE1 <- c(arlE10, round(DxDewma.arl(lambda1, h1, deltas, sided="two", with0=TRUE),
                         digits=2))
arlE20 <- round(xewma.arl(lambda2, h2, 0, sided="two"), digits=2)
arlE2 <- c(arlE20, round(DxDewma.arl(lambda2, h2, deltas, sided="two", with0=TRUE),
                         digits=2))
arlE30 <- round(xewma.arl(lambda3, h3, 0, sided="two"), digits=2)
arlE3 <- c(arlE30, round(DxDewma.arl(lambda3, h3, deltas, sided="two", with0=TRUE),
                         digits=2))
data.frame(delta=c(0, deltas), arlE1, arlE2, arlE3)

## do the same with more digits for the alarm threshold
L0 <- 500
h1 <- xewma.crit(lambda1, L0, sided="two")
h2 <- xewma.crit(lambda2, L0, sided="two")
h3 <- xewma.crit(lambda3, L0, sided="two")
lambdas <- c(lambda1, lambda2, lambda3)
hs <- c(h1, h2, h3) * sqrt(lambdas/(2-lambdas))
hs
# compare with Gan's values 2.204, 1.111, 0.403
round(hs, digits=3)

arlE10 <- round(xewma.arl(lambda1, h1, 0, sided="two"), digits=2)
arlE1 <- c(arlE10, round(DxDewma.arl(lambda1, h1, deltas, sided="two", with0=TRUE),
                         digits=2))
arlE20 <- round(xewma.arl(lambda2, h2, 0, sided="two"), digits=2)
arlE2 <- c(arlE20, round(DxDewma.arl(lambda2, h2, deltas, sided="two", with0=TRUE),
```

```
                         digits=2))
arlE30 <- round(xewma.arl(lambda3, h3, 0, sided="two"), digits=2)
arlE3 <- c(arlE30, round(DxDewma.arl(lambda3, h3, deltas, sided="two", with0=TRUE),
                         digits=2))
data.frame(delta=c(0, deltas), arlE1, arlE2, arlE3)

## Aerne et al. (1991) -- two-sided EWMA
## Table I (continued)
## original numbers are
#     delta  arlE1  arlE2  arlE3
#  0.000000  465.0  465.0  465.0
#  0.005623  77.01  85.93  102.68
#  0.007499  64.61  71.78  85.74
#  0.010000  54.20  59.74  71.22
#  0.013335  45.20  49.58  58.90
#  0.017783  37.76  41.06  48.54
#  0.023714  31.54  33.95  39.87
#  0.031623  26.36  28.06  32.68
#  0.042170  22.06  23.19  26.73
#  0.056234  18.49  19.17  21.84
#  0.074989  15.53  15.87  17.83
#  0.100000  13.07  13.16  14.55
#  0.133352  11.03  10.94  11.88
#  0.177828   9.33   9.12   9.71
#  0.237137   7.91   7.62   7.95
#  0.316228   6.72   6.39   6.52
#  0.421697   5.72   5.38   5.37
#  0.562341   4.88   4.54   4.44
#  0.749894   4.18   3.84   3.68
#  1.000000   3.58   3.27   3.07
#
lambda1 <- .133
lambda2 <- .25
lambda3 <- .5
cE1 <- 2.856
cE2 <- 2.974
cE3 <- 3.049
deltas <- 10^(-(18:0)/8)
arlE10 <- round(xewma.arl(lambda1, cE1, 0, sided="two"), digits=2)
arlE1 <- c(arlE10, round(DxDewma.arl(lambda1, cE1, deltas, sided="two"), digits=2))
arlE20 <- round(xewma.arl(lambda2, cE2, 0, sided="two"), digits=2)
arlE2 <- c(arlE20, round(DxDewma.arl(lambda2, cE2, deltas, sided="two"), digits=2))
arlE30 <- round(xewma.arl(lambda3, cE3, 0, sided="two"), digits=2)
arlE3 <- c(arlE30, round(DxDewma.arl(lambda3, cE3, deltas, sided="two"), digits=2))
data.frame(delta=c(0, round(deltas, digits=6)), arlE1, arlE2, arlE3)


## Fahmy/Elsayed (2006) -- two-sided EWMA
## Table 4 (Monte Carlo results, 10^4 replicates, change point at t=51!)
## original numbers are
#   delta     arl  s.e.
#   0.00  365.749  3.598
#   0.10   12.971  0.029
```

```
#   0.25    7.738  0.015
#   0.50    5.312  0.009
#   0.75    4.279  0.007
#   1.00    3.680  0.006
#   1.25    3.271  0.006
#   1.50    2.979  0.005
#   1.75    2.782  0.004
#   2.00    2.598  0.005
#
lambda <- 0.1
cE <- 2.7
deltas <- c(.1, (1:8)/4)
arlE1 <- c(round(xewma.arl(lambda, cE, 0, sided="two"), digits=3),
           round(DxDewma.arl(lambda, cE, deltas, sided="two"), digits=3))
arlE51 <- c(round(xewma.arl(lambda, cE, 0, sided="two", q=51)[51], digits=3),
     round(DxDewma.arl(lambda, cE, deltas, sided="two", mode="Knoth", q=51),
           digits=3))
data.frame(delta=c(0, deltas), arlE1, arlE51)

## additional Monte Carlo results with 10^8 replicates
#   delta   arl.q=1   s.e.    arl.q=51  s.e.
#   0.00    368.910   0.036   361.714   0.038
#   0.10     12.986   0.000    12.781   0.000
#   0.25      7.758   0.000     7.637   0.000
#   0.50      5.318   0.000     5.235   0.000
#   0.75      4.285   0.000     4.218   0.000
#   1.00      3.688   0.000     3.628   0.000
#   1.25      3.274   0.000     3.233   0.000
#   1.50      2.993   0.000     2.942   0.000
#   1.75      2.808   0.000     2.723   0.000
#   2.00      2.616   0.000     2.554   0.000

## Zou et al. (2009) -- one-sided EWMA
## Table 1
## original values are
#  delta   arl1  arl2  arl3
#  0           ~ 1730
#  0.0005  317   377   440
#  0.001   215   253   297
#  0.005   83.6  92.6  106
#  0.01    55.6  58.8  66.1
#  0.05    22.6  21.1  22.0
#  0.1     15.5  13.9  13.8
#  0.5     6.65  5.56  5.09
#  1.0     4.67  3.83  3.43
#  2.0     3.21  2.74  2.32
#  3.0     2.86  2.06  1.98
#  4.0     2.14  2.00  1.83
l1 <- 0.03479
l2 <- 0.11125
l3 <- 0.23052
c1 <- 2.711
c2 <- 3.033
```

```
c3 <- 3.161
zr <- -6
r  <- 50
deltas <- c(0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1:4)
arl1 <- c(round(xewma.arl(l1, c1, 0, zr=zr, r=r), digits=2),
          round(DxDewma.arl(l1, c1, deltas, zr=zr, r=r), digits=2))
arl2 <- c(round(xewma.arl(l2, c2, 0, zr=zr), digits=2),
          round(DxDewma.arl(l2, c2, deltas, zr=zr, r=r), digits=2))
arl3 <- c(round(xewma.arl(l3, c3, 0, zr=zr, r=r), digits=2),
          round(DxDewma.arl(l3, c3, deltas, zr=zr, r=r), digits=2))
data.frame(delta=c(0, deltas), arl1, arl2, arl3)

## End(Not run)
```

xDgrsr.arl — *Compute ARLs of Shiryaev-Roberts schemes under drift*

Description

Computation of the (zero-state and other) Average Run Length (ARL) under drift for Shiryaev-Roberts schemes monitoring normal mean.

Usage

```
xDgrsr.arl(k, g, delta, zr = 0, hs = NULL, sided = "one", m = NULL,
mode = "Gan", q = 1, r = 30, with0 = FALSE)
```

Arguments

`k`	reference value of the Shiryaev-Roberts scheme.
`g`	control limit (alarm threshold) of Shiryaev-Roberts scheme.
`delta`	true drift parameter.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided Shiryaev-Roberts schemes by choosing `"one"` and `"two"`, respectively. Currentlly, the two-sided scheme is not implemented.
`m`	parameter used if `mode="Gan"`. `m` is design parameter of Gan's approach. If `m=NULL`, then `m` will increased until the resulting ARL does not change anymore.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu! = 0$ conditional delays, that is, $E_q(L - q + 1 \| L \geq)$, will be determined. Note that mu0=0 is implicitely fixed. Deploy large `q` to mimic steady-state. It works only for `mode="Knoth"`.
`mode`	decide whether Gan's or Knoth's approach is used. Use `"Gan"` and `"Knoth"`, respectively. `"Knoth"` is not implemented yet.

r
: number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

with0
: defines whether the first observation used for the RL calculation follows already 1*delta or still 0*delta. With `q` additional flexibility is given.

Details

Based on Gan (1991) or Knoth (2003), the ARL is calculated for Shiryaev-Roberts schemes under drift. In case of Gan's framework, the usual ARL function with mu=m*delta is determined and recursively via m-1, m-2, ... 1 (or 0) the drift ARL determined. The framework of Knoth allows to calculate ARLs for varying parameters, such as control limits and distributional parameters. For details see the cited papers.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

F. F. Gan (1991), EWMA control chart under linear drift, *J. Stat. Comput. Simulation 38*, 181-200.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2012), More on Control Charting under Drift, in: *Frontiers in Statistical Quality Control 10*, H.-J. Lenz, W. Schmid and P.-T. Wilrich (Eds.), Physica Verlag, Heidelberg, Germany, 53-68.

C. Zou, Y. Liu and Z. Wang (2009), Comparisons of control schemes for monitoring the means of processes subject to drifts, *Metrika 70*, 141-163.

See Also

`xewma.arl` and `xewma.ad` for zero-state and steady-state ARL computation of EWMA control charts for the classical step change model.

Examples

```
## Not run:
## Monte Carlo example with 10^8 replicates
#   delta      arl    s.e.
#   0.0001 381.8240   0.0304
#   0.0005 238.4630   0.0148
#   0.001  177.4061   0.0097
#   0.002  125.9055   0.0061
#   0.005   75.7574   0.0031
#   0.01    50.2203   0.0018
#   0.02    32.9458   0.0011
#   0.05    18.9213   0.0005
```

```
#   0.1     12.6054   0.0003
#   0.5      5.2157   0.0001
#   1        3.6537   0.0001
#   3        2.0289   0.0000
k <- .5
L0 <- 500
zr <- -7
r <- 50
g <- xgrsr.crit(k, L0, zr=zr, r=r)
DxDgrsr.arl <- Vectorize(xDgrsr.arl, "delta")
deltas <- c(0.0001, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.5, 1, 3)
arls <- round(DxDgrsr.arl(k, g, deltas, zr=zr, r=r), digits=4)
data.frame(deltas, arls)

## End(Not run)
```

---

xDshewhartrunsrules.arl

Compute ARLs of Shewhart control charts with and without runs rules under drift

---

Description

Computation of the zero-state Average Run Length (ARL) under drift for Shewhart control charts with and without runs rules monitoring normal mean.

Usage

```
xDshewhartrunsrules.arl(delta, c = 1, m = NULL, type = "12")

xDshewhartrunsrulesFixedm.arl(delta, c = 1, m = 100, type = "12")
```

Arguments

`delta`	true drift parameter.
`c`	normalizing constant to ensure specific alarming behavior.
`type`	controls the type of Shewhart chart used, seed details section.
`m`	parameter of Gan's approach. If `m=NULL`, then `m` will increased until the resulting ARL does not change anymore.

Details

Based on Gan (1991), the ARL is calculated for Shewhart control charts with and without runs rules under drift. The usual ARL function with mu=m*delta is determined and recursively via m-1, m-2, ... 1 (or 0) the drift ARL determined. `xDshewhartrunsrulesFixedm.arl` is the actual work horse, while `xDshewhartrunsrules.arl` provides a convenience wrapper. Note that Aerne et al. (1991) deployed a method that is quite similar to Gan's algorithm. For `type` see the help page of `xshewhartrunsrules.arl`.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

F. F. Gan (1991), EWMA control chart under linear drift, *J. Stat. Comput. Simulation 38*, 181-200.

L. A. Aerne, C. W. Champ and S. E. Rigdon (1991), Evaluation of control charts under linear trend, *Commun. Stat., Theory Methods 20*, 3341-3349.

See Also

`xshewhartrunsrules.arl` for zero-state ARL computation of Shewhart control charts with and without runs rules for the classical step change model.

Examples

```
## Aerne et al. (1991)
## Table I (continued)
## original numbers are
#     delta arl1of1 arl2of3 arl4of5  arl10
#  0.005623  136.67  120.90  105.34 107.08
#  0.007499  114.98  101.23   88.09  89.94
#  0.010000   96.03   84.22   73.31  75.23
#  0.013335   79.69   69.68   60.75  62.73
#  0.017783   65.75   57.38   50.18  52.18
#  0.023714   53.99   47.06   41.33  43.35
#  0.031623   44.15   38.47   33.99  36.00
#  0.042170   35.97   31.36   27.91  29.90
#  0.056234   29.21   25.51   22.91  24.86
#  0.074989   23.65   20.71   18.81  20.70
#  0.100000   19.11   16.79   15.45  17.29
#  0.133352   15.41   13.61   12.72  14.47
#  0.177828   12.41   11.03   10.50  12.14
#  0.237137    9.98    8.94    8.71  10.18
#  0.316228    8.02    7.25    7.26   8.45
#  0.421697    6.44    5.89    6.09   6.84
#  0.562341    5.17    4.80    5.15   5.48
#  0.749894    4.16    3.92    4.36   4.39
#  1.000000    3.35    3.22    3.63   3.52
c1of1 <- 3.069/3
c2of3 <- 2.1494/2
c4of5 <- 1.14
c10   <- 3.2425/3
DxDshewhartrunsrules.arl <- Vectorize(xDshewhartrunsrules.arl, "delta")
deltas <- 10^(-(18:0)/8)
arl1of1 <-
round(DxDshewhartrunsrules.arl(deltas, c=c1of1, type="1"), digits=2)
```

```
arl2of3 <-
round(DxDshewhartrunsrules.arl(deltas, c=c2of3, type="12"), digits=2)
arl4of5 <-
round(DxDshewhartrunsrules.arl(deltas, c=c4of5, type="13"), digits=2)
arl10 <-
round(DxDshewhartrunsrules.arl(deltas, c=c10, type="SameSide10"), digits=2)
data.frame(delta=round(deltas, digits=6), arl1of1, arl2of3, arl4of5, arl10)
```

`xewma.ad` *Compute steady-state ARLs of EWMA control charts*

Description

Computation of the steady-state Average Run Length (ARL) for different types of EWMA control charts monitoring normal mean.

Usage

```
xewma.ad(l, c, mu1, mu0=0, zr=0, z0=0, sided="one", limits="fix",
steady.state.mode="conditional", r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu1`	out-of-control mean.
`mu0`	in-control mean.
`zr`	reflection border for the one-sided chart.
`z0`	restarting value of the EWMA sequence in case of a false alarm in `steady.state.mode="cyclical"`.
`sided`	distinguishes between one- and two-sided two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`steady.state.mode`	distinguishes between two steady-state modes – conditional and cyclical.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

Details

`xewma.ad` determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature and using the power method for deriving the largest in magnitude eigenvalue and the related left eigenfunction.

Value

Returns a single value which resembles the steady-state ARL.

Author(s)

Sven Knoth

References

R. B. Crosier (1986), A new two-sided cumulative quality control scheme, *Technometrics 28*, 187-194.

S. V. Crowder (1987), A simple method for studying run-length distributions of exponentially weighted moving average charts, *Technometrics 29*, 401-407.

J. M. Lucas and M. S. Saccucci (1990), Exponentially weighted moving average control schemes: Properties and enhancements, *Technometrics 32*, 1-12.

See Also

`xewma.arl` for zero-state ARL computation and `xcusum.ad` for the steady-state ARL of CUSUM control charts.

Examples

```
## comparison of zero-state (= worst case ) and steady-state performance
## for two-sided EWMA control charts

l <- .1
c <- xewma.crit(l,500,sided="two")
mu <- c(0,.5,1,1.5,2)
arl <- sapply(mu,l=l,c=c,sided="two",xewma.arl)
ad <- sapply(mu,l=l,c=c,sided="two",xewma.ad)
round(cbind(mu,arl,ad),digits=2)

## Lucas/Saccucci (1990)
## two-sided EWMA

## with fixed limits
l1 <- .5
l2 <- .03
c1 <- 3.071
c2 <- 2.437
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,3.5,4,5)
ad1 <- sapply(mu,l=l1,c=c1,sided="two",xewma.ad)
ad2 <- sapply(mu,l=l2,c=c2,sided="two",xewma.ad)
round(cbind(mu,ad1,ad2),digits=2)

## original results are (in Table 3)
## 0.00 499.   480.
## 0.25 254.    74.1
## 0.50  88.4   28.6
```

```
## 0.75  35.7   17.3
## 1.00  17.3   12.5
## 1.50   6.44   8.00
## 2.00   3.58   5.95
## 2.50   2.47   4.78
## 3.00   1.91   4.02
## 3.50   1.58   3.49
## 4.00   1.36   3.09
## 5.00   1.10   2.55
```

`xewma.arl` *Compute ARLs of EWMA control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts monitoring normal mean.

Usage

```
xewma.arl(l,c,mu,zr=0,hs=0,sided="one",limits="fix",q=1,r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu`	true mean.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu! = 0$ conditional delays, that is, $E_q(L - q + 1 \mid L \geq q)$, will be determined. Note that mu0=0 is implicitely fixed.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

Details

In case of the EWMA chart with fixed control limits, `xewma.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature. If `limits` is not `"fix"`, then the method presented in Knoth (2003) is utilized. Note that for one-sided EWMA charts (`sided="one"`), only `"vacl"` and `"stat"` are deployed, while for two-sided ones (`sided="two"`) also `"fir"`, `"both"` (combination of `"fir"` and `"vacl"`), and `"Steiner"` are implemented. For details see Knoth (2004).

Value

Except for the fixed limits EWMA charts it returns a single value which resembles the ARL. For fixed limits charts, it returns a vector of length q which resembles the ARL and the sequence of conditional expected delays for q=1 and q>1, respectively.

Author(s)

Sven Knoth

References

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

S. V. Crowder (1987), A simple method for studying run-length distributions of exponentially weighted moving average charts, *Technometrics 29*, 401-407.

J. M. Lucas and M. S. Saccucci (1990), Exponentially weighted moving average control schemes: Properties and enhancements, *Technometrics 32*, 1-12.

S. Chandrasekaran, J. R. English and R. L. Disney (1995), Modeling and analysis of EWMA control schemes with variance-adjusted control limits, *IIE Transactions 277*, 282-290.

T. R. Rhoads, D. C. Montgomery and C. M. Mastrangelo (1996), Fast initial response scheme for exponentially weighted moving average control chart, *Quality Engineering 9*, 317-327.

S. H. Steiner (1999), EWMA control charts with time-varying control limits and fast initial response, *Journal of Quality Technology 31*, 75-86.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

See Also

`xcusum.arl` for zero-state ARL computation of CUSUM control charts and `xewma.ad` for the steady-state ARL.

Examples

```
## Waldmann (1986), one-sided EWMA
l <- .75
round(xewma.arl(l,2*sqrt((2-l)/l),0,zr=-4*sqrt((2-l)/l)),digits=1)
l <- .5
round(xewma.arl(l,2*sqrt((2-l)/l),0,zr=-4*sqrt((2-l)/l)),digits=1)
## original values are 209.3 and 3907.5 (in Table 2).

## Waldmann (1986), two-sided EWMA with fixed control limits
l <- .75
round(xewma.arl(l,2*sqrt((2-l)/l),0,sided="two"),digits=1)
l <- .5
round(xewma.arl(l,2*sqrt((2-l)/l),0,sided="two"),digits=1)
## original values are 104.0 and 1952 (in Table 1).
```

```
## Crowder (1987), two-sided EWMA with fixed control limits
l1 <- .5
l2 <- .05
c <- 2
mu <- (0:16)/4
arl1 <- sapply(mu,l=l1,c=c,sided="two",xewma.arl)
arl2 <- sapply(mu,l=l2,c=c,sided="two",xewma.arl)
round(cbind(mu,arl1,arl2),digits=2)

## original results are (in Table 1)
## 0.00 26.45 127.53
## 0.25 20.12  43.94
## 0.50 11.89  18.97
## 0.75  7.29  11.64
## 1.00  4.91   8.38
## 1.25  3.95*  6.56
## 1.50  2.80   5.41
## 1.75  2.29   4.62
## 2.00  1.94   4.04
## 2.25  1.70   3.61
## 2.50  1.51   3.26
## 2.75  1.37   2.99
## 3.00  1.26   2.76
## 3.25  1.18   2.56
## 3.50  1.12   2.39
## 3.75  1.08   2.26
## 4.00  1.05   2.15  (* -- in Crowder (1987) typo!?)

## Lucas/Saccucci (1990)
## two-sided EWMA

## with fixed limits
l1 <- .5
l2 <- .03
c1 <- 3.071
c2 <- 2.437
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,3.5,4,5)
arl1 <- sapply(mu,l=l1,c=c1,sided="two",xewma.arl)
arl2 <- sapply(mu,l=l2,c=c2,sided="two",xewma.arl)
round(cbind(mu,arl1,arl2),digits=2)

## original results are (in Table 3)
## 0.00 500.   500.
## 0.25 255.    76.7
## 0.50  88.8   29.3
## 0.75  35.9   17.6
## 1.00  17.5   12.6
## 1.50   6.53   8.07
## 2.00   3.63   5.99
## 2.50   2.50   4.80
## 3.00   1.93   4.03
## 3.50   1.58   3.49
```

```
## 4.00   1.34   3.11
## 5.00   1.07   2.55

## Not run:
## with fir feature
l1 <- .5
l2 <- .03
c1 <- 3.071
c2 <- 2.437
hs1 <- c1/2
hs2 <- c2/2
mu <- c(0,.5,1,2,3,5)
arl1 <- sapply(mu,l=l1,c=c1,hs=hs1,sided="two",limits="fir",xewma.arl)
arl2 <- sapply(mu,l=l2,c=c2,hs=hs2,sided="two",limits="fir",xewma.arl)
round(cbind(mu,arl1,arl2),digits=2)

## original results are (in Table 5)
## 0.0 487.   406.
## 0.5  86.1   18.4
## 1.0  15.9    7.36
## 2.0   2.87   3.43
## 3.0   1.45   2.34
## 5.0   1.01   1.57

## Chandrasekaran, English, Disney (1995)
## two-sided EWMA with fixed and variance adjusted limits (vacl)

l1 <- .25
l2 <- .1
c1s <- 2.9985
c1n <- 3.0042
c2s <- 2.8159
c2n <- 2.8452
mu <- c(0,.25,.5,.75,1,2)
arl1s <- sapply(mu,l=l1,c=c1s,sided="two",xewma.arl)
arl1n <- sapply(mu,l=l1,c=c1n,sided="two",limits="vacl",xewma.arl)
arl2s <- sapply(mu,l=l2,c=c2s,sided="two",xewma.arl)
arl2n <- sapply(mu,l=l2,c=c2n,sided="two",limits="vacl",xewma.arl)
round(cbind(mu,arl1s,arl1n,arl2s,arl2n),digits=2)

## original results are (in Table 2)
## 0.00 500.   500.   500.   500.
## 0.25 170.09 167.54 105.90  96.6
## 0.50  48.14  45.65  31.08  24.35
## 0.75  20.02  19.72  15.71  10.74
## 1.00  11.07   9.37  10.23   6.35
## 2.00   3.59   2.64   4.32   2.73

## The results in Chandrasekaran, English, Disney (1995) are not
## that accurate. Let us consider the more appropriate comparison

c1s <- xewma.crit(l1,500,sided="two")
c1n <- xewma.crit(l1,500,sided="two",limits="vacl")
```

```
c2s <- xewma.crit(l2,500,sided="two")
c2n <- xewma.crit(l2,500,sided="two",limits="vacl")
mu <- c(0,.25,.5,.75,1,2)
arl1s <- sapply(mu,l=l1,c=c1s,sided="two",xewma.arl)
arl1n <- sapply(mu,l=l1,c=c1n,sided="two",limits="vacl",xewma.arl)
arl2s <- sapply(mu,l=l2,c=c2s,sided="two",xewma.arl)
arl2n <- sapply(mu,l=l2,c=c2n,sided="two",limits="vacl",xewma.arl)
round(cbind(mu,arl1s,arl1n,arl2s,arl2n),digits=2)

## which demonstrate the abilities of the variance-adjusted limits
## scheme more explicitely.

## Rhoads, Montgomery, Mastrangelo (1996)
## two-sided EWMA with fixed and variance adjusted limits (vacl),
## with fir and both features

l <- .03
c <- 2.437
mu <- c(0,.5,1,1.5,2,3,4)
sl <- sqrt(l*(2-l))
arlfix  <- sapply(mu,l=l,c=c,sided="two",xewma.arl)
arlvacl <- sapply(mu,l=l,c=c,sided="two",limits="vacl",xewma.arl)
arlfir  <- sapply(mu,l=l,c=c,hs=c/2,sided="two",limits="fir",xewma.arl)
arlboth <- sapply(mu,l=l,c=c,hs=c/2*sl,sided="two",limits="both",xewma.arl)
round(cbind(mu,arlfix,arlvacl,arlfir,arlboth),digits=1)

## original results are (in Table 1)
## 0.0 477.3* 427.9* 383.4* 286.2*
## 0.5  29.7   20.0   18.6   12.8
## 1.0  12.5    6.5    7.4    3.6
## 1.5   8.1    3.3    4.6    1.9
## 2.0   6.0    2.2    3.4    1.4
## 3.0   4.0    1.3    2.4    1.0
## 4.0   3.1    1.1    1.9    1.0
## * -- the in-control values differ sustainably from the true values!

## Steiner (1999)
## two-sided EWMA control charts with various modifications

## fixed vs. variance adjusted limits

l <- .05
c <- 3
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,3.5,4)
arlfix <- sapply(mu,l=l,c=c,sided="two",xewma.arl)
arlvacl <- sapply(mu,l=l,c=c,sided="two",limits="vacl",xewma.arl)
round(cbind(mu,arlfix,arlvacl),digits=1)

## original results are (in Table 2)
## 0.00 1379.0   1353.0
## 0.25  135.0    127.0
## 0.50   37.4     32.5
## 0.75   20.0     15.6
```

```
## 1.00   13.5      9.0
## 1.50    8.3      4.5
## 2.00    6.0      2.8
## 2.50    4.8      2.0
## 3.00    4.0      1.6
## 3.50    3.4      1.3
## 4.00    3.0      1.1

## fir, both, and Steiner's modification

l <- .03
cfir <- 2.44
cboth <- 2.54
cstein <- 2.55
hsfir <- cfir/2
hsboth <- cboth/2*sqrt(l*(2-l))
mu <- c(0,.5,1,1.5,2,3,4)
arlfir <- sapply(mu,l=l,c=cfir,hs=hsfir,sided="two",limits="fir",xewma.arl)
arlboth <- sapply(mu,l=l,c=cboth,hs=hsboth,sided="two",limits="both",xewma.arl)
arlstein <- sapply(mu,l=l,c=cstein,sided="two",limits="Steiner",xewma.arl)
round(cbind(mu,arlfir,arlboth,arlstein),digits=1)

## original values are (in Table 5)
## 0.0 383.0   384.0   391.0
## 0.5  18.6    14.9    13.8
## 1.0   7.4     3.9     3.6
## 1.5   4.6     2.0     1.8
## 2.0   3.4     1.4     1.3
## 3.0   2.4     1.1     1.0
## 4.0   1.9     1.0     1.0

## SAS/QC manual 1999
## two-sided EWMA control charts with fixed limits

l <- .25
c <- 3
mu <- 1
print(xewma.arl(l,c,mu,sided="two"),digits=11)

# original value is 11.154267016.

## Some recent examples for one-sided EWMA charts
## with varying limits and in the so-called stationary mode

# 1. varying limits = "vacl"

lambda <- .1
L0 <- 500

## Monte Carlo results (10^9 replicates)
# mu    ARL      s.e.
# 0     500.00   0.0160
# 0.5   21.637   0.0006
```

```
# 1     6.7596   0.0001
# 1.5   3.5398   0.0001
# 2     2.3038   0.0000
# 2.5   1.7004   0.0000
# 3     1.3675   0.0000

zr <- -6
r <- 50
c <- xewma.crit(lambda, L0, zr=zr, limits="vacl", r=r)
Mxewma.arl <- Vectorize(xewma.arl, "mu")
mus <- (0:6)/2
arls <- round(Mxewma.arl(lambda, c, mus, zr=zr, limits="vacl", r=r), digits=4)
data.frame(mus, arls)

# 2. stationary mode, i. e. limits = "stat"

## Monte Carlo results (10^9 replicates)
# mu    ARL      s.e.
# 0     500.00   0.0159
# 0.5   22.313   0.0006
# 1     7.2920   0.0001
# 1.5   3.9064   0.0001
# 2     2.5131   0.0000
# 2.5   1.7983   0.0000
# 3     1.4029   0.0000

c <- xewma.crit(lambda, L0, zr=zr, limits="stat", r=r)
arls <- round(Mxewma.arl(lambda, c, mus, zr=zr, limits="stat", r=r), digits=4)
data.frame(mus, arls)

## End(Not run)
```

`xewma.arl.f` *Compute ARL function of EWMA control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) function for different types of EWMA control charts monitoring normal mean.

Usage

```
xewma.arl.f(l,c,mu,zr=0,sided="one",limits="fix",r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu`	true mean.

`zr`	reflection border for the one-sided chart.
`sided`	distinguishes between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

Details

It is a convenience function to yield the ARL as function of the head start `hs`. For more details see `xewma.arl`.

Value

It returns a function of a single argument, `hs=x` which maps the head-start value `hs` to the ARL.

Author(s)

Sven Knoth

References

S. V. Crowder (1987), A simple method for studying run-length distributions of exponentially weighted moving average charts, *Technometrics 29*, 401-407.

See Also

`xewma.arl` for zero-state ARL for one specific head-start `hs`.

Examples

```
# will follow
```

`xewma.arl.prerun` *Compute ARLs of EWMA control charts in case of estimated parameters*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts monitoring normal mean if the in-control mean, standard deviation, or both are estimated by a pre run.

Usage

```
xewma.arl.prerun(l, c, mu, zr=0, hs=0, sided="two", limits="fix", q=1,
size=100, df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10)

xewma.crit.prerun(l, L0, mu, zr=0, hs=0, sided="two", limits="fix", size=100,
df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10,
c.error=1e-12, L.error=1e-9, OUTPUT=FALSE)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu`	true mean shift.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (give fast initial response).
`sided`	distinguish between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguish between different control limits behavior.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu != 0$ conditional delays, that is, $E_q(L - q + 1 \mid L \geq q)$, will be determined. Note that mu0=0 is implicitely fixed.
`size`	pre run sample size.
`df`	Degrees of freedom of the pre run variance estimator. Typically it is simply `size` - 1. If the pre run is collected in batches, then also other values are needed.
`estimated`	name the parameter to be estimated within the `"mu"`, `"sigma"`, `"both"`.
`qm.mu`	number of quadrature nodes for convoluting the mean uncertainty.
`qm.sigma`	number of quadrature nodes for convoluting the standard deviation uncertainty.
`truncate`	size of truncated tail.
`L0`	in-control ARL.
`c.error`	error bound for two succeeding values of the critical value during applying the secant rule.
`L.error`	error bound for the ARL level `L0` during applying the secant rule.
`OUTPUT`	activate or deactivate additional output.

Details

Essentially, the ARL function `xewma.arl` is convoluted with the distribution of the sample mean, standard deviation or both. For details see Jones/Champ/Rigdon (2001) and Knoth (2014?).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

L. A. Jones, C. W. Champ, S. E. Rigdon (2001), The performance of exponentially weighted moving average charts with estimated parameters, *Technometrics 43*, 156-167.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

S. Knoth (2014?), tbd, *tbd*, tbd-tbd.

See Also

`xewma.arl` for the usual zero-state ARL computation.

Examples

```
## Jones/Champ/Rigdon (2001)

c4m <- function(m, n) sqrt(2)*gamma( (m*(n-1)+1)/2 )/sqrt( m*(n-1) )/gamma( m*(n-1)/2 )

n <- 5 # sample size
m <- 20 # pre run with 20 samples of size n = 5
C4m <- c4m(m, n) # needed for bias correction

# Table 1, 3rd column
lambda <- 0.2
L <- 2.636

xewma.ARL <- Vectorize("xewma.arl", "mu")
xewma.ARL.prerun <- Vectorize("xewma.arl.prerun", "mu")

mu <- c(0, .25, .5, 1, 1.5, 2)
ARL <- round(xewma.ARL(lambda, L, mu, sided="two"), digits=2)
p.ARL <- round(xewma.ARL.prerun(lambda, L/C4m, mu, sided="two",
size=m, df=m*(n-1), estimated="both", qm.mu=70), digits=2)

# Monte-Carlo with 10^8 repetitions: 200.325 (0.020) and 144.458 (0.022)
cbind(mu, ARL, p.ARL)

## Not run:
# Figure 5, subfigure r = 0.2
mu_ <- (0:85)/40
ARL_ <- round(xewma.ARL(lambda, L, mu_, sided="two"), digits=2)
p.ARL_ <- round(xewma.ARL.prerun(lambda, L/C4m, mu_, sided="two",
size=m, df=m*(n-1), estimated="both"), digits=2)

plot(mu_, ARL_, type="l", xlab=expression(delta), ylab="ARL", xlim=c(0,2))
```

```
abline(v=0, h=0, col="grey", lwd=.7)
points(mu, ARL, pch=5)
lines(mu_, p.ARL_, col="blue")
points(mu, p.ARL, pch=18, col ="blue")
legend("topright", c("Known", "Estimated"), col=c("black", "blue"),
lty=1, pch=c(5, 18))

## End(Not run)
```

`xewma.crit` *Compute critical values of EWMA control charts*

Description

Computation of the critical values (similar to alarm limits) for different types of EWMA control charts monitoring normal mean.

Usage

```
xewma.crit(l,L0,mu0=0,zr=0,hs=0,sided="one",limits="fix",r=40,c0=NULL)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`L0`	in-control ARL.
`mu0`	in-control mean.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).
`c0`	starting value for iteration rule.

Details

`xewma.crit` determines the critical values (similar to alarm limits) for given in-control ARL `L0` by applying secant rule and using `xewma.arl()`.

Value

Returns a single value which resembles the critical value `c`.

Author(s)

Sven Knoth

References

S. V. Crowder (1989), Design of exponentially weighted moving average schemes, *Journal of Quality Technology 21*, 155-162.

See Also

`xewma.arl` for zero-state ARL computation.

Examples

```
l <- .1
incontrolARL <- c(500,5000,50000)
sapply(incontrolARL,l=l,sided="two",xewma.crit,r=35) # accuracy with 35 nodes
sapply(incontrolARL,l=l,sided="two",xewma.crit)      # accuracy with 40 nodes
sapply(incontrolARL,l=l,sided="two",xewma.crit,r=50) # accuracy with 50 nodes

## Crowder (1989)
## two-sided EWMA control charts with fixed limits

l <- c(.05,.1,.15,.2,.25)
L0 <- 250
round(sapply(l,L0=L0,sided="two",xewma.crit),digits=2)

## original values are 2.32, 2.55, 2.65, 2.72, and 2.76.
```

`xewma.q` *Compute RL quantiles of EWMA control charts*

Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal mean.

Usage

```
xewma.q(l, c, mu, alpha, zr=0, hs=0, sided="two", limits="fix", q=1, r=40)

xewma.q.crit(l, L0, mu, alpha, zr=0, hs=0, sided="two", limits="fix", r=40,
c.error=1e-12, a.error=1e-9, OUTPUT=FALSE)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu`	true mean.
`alpha`	quantile level.
`zr`	reflection border for the one-sided chart.

hs	so-called headstart (enables fast initial response).
sided	distinguishes between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
limits	distinguishes between different control limits behavior.
q	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu ! = 0$ conditional delays, that is, $E_q(L - q + 1 \| L \geq)$, will be determined. Note that mu0=0 is implicitely fixed.
r	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).
L0	in-control quantile value.
c.error	error bound for two succeeding values of the critical value during applying the secant rule.
a.error	error bound for the quantile level `alpha` during applying the secant rule.
OUTPUT	activate or deactivate additional output.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure. If `limits` is not `"fix"`, then the method presented in Knoth (2003) is utilized. Note that for one-sided EWMA charts (`sided="one"`), only `"vacl"` and `"stat"` are deployed, while for two-sided ones (`sided="two"`) also `"fir"`, `"both"` (combination of `"fir"` and `"vacl"`), and `"Steiner"` are implemented. For details see Knoth (2004).

Value

Returns a single value which resembles the RL quantile of order `q`.

Author(s)

Sven Knoth

References

F. F. Gan (1993), An optimal design of EWMA control charts based on the median run length, *J. Stat. Comput. Simulation 45*, 169-184.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

S. Knoth (2015), Run length quantiles of EWMA control charts monitoring normal mean or/and variance, *International Journal of Production Research 53*, 4629-4647.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts.

Examples

```
## Gan (1993), two-sided EWMA with fixed control limits
## some values of his Table 1 -- any median RL should be 500
XEWMA.Q <- Vectorize("xewma.q", c("l", "c"))
G.lambda <- c(.05, .1, .15, .2, .25)
G.h <- c(.441, .675, .863, 1.027, 1.177)
MEDIAN <- ceiling(XEWMA.Q(G.lambda, G.h/sqrt(G.lambda/(2-G.lambda)),
0, .5, sided="two"))
print(cbind(G.lambda, MEDIAN))

## increase accuracy of thresholds

# (i) calculate threshold for given in-control median value by
#     deplyoing secant rule
XEWMA.q.crit <- Vectorize("xewma.q.crit", "l")

# (ii) re-calculate the thresholds and remove the standardization step
L0 <- 500
G.h.new <- XEWMA.q.crit(G.lambda, L0, 0, .5, sided="two")
G.h.new <- round(G.h.new * sqrt(G.lambda/(2-G.lambda)), digits=5)

# (iii) compare Gan's original values and the new ones with 5 digits
print(cbind(G.lambda, G.h.new, G.h))

# (iv) calculate the new medians
MEDIAN <- ceiling(XEWMA.Q(G.lambda, G.h.new/sqrt(G.lambda/(2-G.lambda)),
0, .5, sided="two"))
print(cbind(G.lambda, MEDIAN))
```

---

xewma.q.prerun — *Compute RL quantiles of EWMA control charts in case of estimated parameters*

Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal mean if the in-control mean, standard deviation, or both are estimated by a pre run.

Usage

```
xewma.q.prerun(l, c, mu, p, zr=0, hs=0, sided="two", limits="fix", q=1, size=100,
df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10, bound=1e-10)

xewma.q.crit.prerun(l, L0, mu, p, zr=0, hs=0, sided="two", limits="fix", size=100,
df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10, bound=1e-10,
c.error=1e-10, p.error=1e-9, OUTPUT=FALSE)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu`	true mean shift.
`p`	quantile level.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (give fast initial response).
`sided`	distinguish between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguish between different control limits behavior.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu != 0$ conditional delays, that is, $E_q(L - q + 1 \| L \geq)$, will be determined. Note that mu0=0 is implicitely fixed.
`size`	pre run sample size.
`df`	Degrees of freedom of the pre run variance estimator. Typically it is simply `size - 1`. If the pre run is collected in batches, then also other values are needed.
`estimated`	name the parameter to be estimated within the `"mu"`, `"sigma"`, `"both"`.
`qm.mu`	number of quadrature nodes for convoluting the mean uncertainty.
`qm.sigma`	number of quadrature nodes for convoluting the standard deviation uncertainty.
`truncate`	size of truncated tail.
`bound`	control when the geometric tail kicks in; the larger the quicker and less accurate; `bound` should be larger than 0 and less than 0.001.
`L0`	in-control quantile value.
`c.error`	error bound for two succeeding values of the critical value during applying the secant rule.
`p.error`	error bound for the quantile level `p` during applying the secant rule.
`OUTPUT`	activate or deactivate additional output.

Details

Essentially, the ARL function `xewma.q` is convoluted with the distribution of the sample mean, standard deviation or both. For details see Jones/Champ/Rigdon (2001) and Knoth (2014?).

Value

Returns a single value which resembles the RL quantile of order `q`.

Author(s)

Sven Knoth

References

L. A. Jones, C. W. Champ, S. E. Rigdon (2001), The performance of exponentially weighted moving average charts with estimated parameters, *Technometrics 43*, 156-167.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

S. Knoth (2014?), tbd, *tbd*, tbd-tbd.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`xewma.q` for the usual RL quantiles computation of EWMA control charts.

Examples

```
## Jones/Champ/Rigdon (2001)

c4m <- function(m, n) sqrt(2)*gamma( (m*(n-1)+1)/2 )/sqrt( m*(n-1) )/gamma( m*(n-1)/2 )

n <- 5 # sample size
m <- 20 # pre run with 20 samples of size n = 5
C4m <- c4m(m, n) # needed for bias correction

# Table 1, 3rd column
lambda <- 0.2
L <- 2.636

xewma.Q <- Vectorize("xewma.q", "mu")
xewma.Q.prerun <- Vectorize("xewma.q.prerun", "mu")

mu <- c(0, .25, .5, 1, 1.5, 2)
Q1  <- ceiling(xewma.Q(lambda, L, mu, 0.1, sided="two"))
Q2  <- ceiling(xewma.Q(lambda, L, mu, 0.5, sided="two"))
Q3  <- ceiling(xewma.Q(lambda, L, mu, 0.9, sided="two"))

cbind(mu, Q1, Q2, Q3)

## Not run:
p.Q1 <- xewma.Q.prerun(lambda, L/C4m, mu, 0.1, sided="two",
size=m, df=m*(n-1), estimated="both")
p.Q2 <- xewma.Q.prerun(lambda, L/C4m, mu, 0.5, sided="two",
size=m, df=m*(n-1), estimated="both")
p.Q3 <- xewma.Q.prerun(lambda, L/C4m, mu, 0.9, sided="two",
size=m, df=m*(n-1), estimated="both")

cbind(mu, p.Q1, p.Q2, p.Q3)

## End(Not run)
```

```
## original values are
#    mu Q1  Q2  Q3 p.Q1 p.Q2 p.Q3
#  0.00 25 140 456   13   73  345
#  0.25 12  56 174    9   46  253
#  0.50  7  20  56    6   20  101
#  1.00  4   7  15    3    7   18
#  1.50  3   4   7    2    4    8
#  2.00  2   3   5    2    3    5
```

---

xewma.sf *Compute the survival function of EWMA run length*

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal mean.

Usage

```
xewma.sf(l, c, mu, n, zr=0, hs=0, sided="one", limits="fix", q=1, r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu`	true mean.
`n`	calculate sf up to value `n`.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state situation for the in-control and out-of-control case, respectively, are calculated. Note that mu0=0 is implicitely fixed.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

Details

The survival function P(L>n) and derived from it also the cdf P(L<=n) and the pmf P(L=n) illustrate the distribution of the EWMA run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann's survival function iteration procedure. For varying limits and for change points after 1 the algorithm from Knoth (2004) is applied. Note that for one-sided EWMA charts (`sided="one"`), only `"vacl"` and `"stat"` are deployed, while for two-sided ones (`sided="two"`) also `"fir"`, `"both"` (combination of `"fir"` and `"vacl"`), and `"Steiner"` are implemented. For details see Knoth (2004).

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

F. F. Gan (1993), An optimal design of EWMA control charts based on the median run length, *J. Stat. Comput. Simulation 45*, 169-184.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts.

Examples

```
## Gan (1993), two-sided EWMA with fixed control limits
## some values of his Table 1 -- any median RL should be 500

G.lambda <- c(.05, .1, .15, .2, .25)
G.h <- c(.441, .675, .863, 1.027, 1.177)/sqrt(G.lambda/(2-G.lambda))

for ( i in 1:length(G.lambda) ) {
  SF <- xewma.sf(G.lambda[i], G.h[i], 0, 1000)
  if (i==1) plot(1:length(SF), SF, type="l", xlab="n", ylab="P(L>n)")
  else lines(1:length(SF), SF, col=i)
}
```

`xewma.sf.prerun` *Compute the survival function of EWMA run length in case of estimated parameters*

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal mean if the in-control mean, standard deviation, or both are estimated by a pre run.

Usage

```
xewma.sf.prerun(l, c, mu, n, zr=0, hs=0, sided="one", limits="fix", q=1,
size=100, df=NULL, estimated="mu", qm.mu=30, qm.sigma=30,
truncate=1e-10, tail_approx=TRUE, bound=1e-10)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`mu`	true mean.
`n`	calculate sf up to value `n`.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (give fast initial response).
`sided`	distinguish between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguish between different control limits behavior.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state situation for the in-control and out-of-control case, respectively, are calculated. Note that mu0=0 is implicitely fixed.
`size`	pre run sample size.
`df`	degrees of freedom of the pre run variance estimator. Typically it is simply `size` - 1. If the pre run is collected in batches, then also other values are needed.
`estimated`	name the parameter to be estimated within the `"mu"`, `"sigma"`, `"both"`.
`qm.mu`	number of quadrature nodes for convoluting the mean uncertainty.
`qm.sigma`	number of quadrature nodes for convoluting the standard deviation uncertainty.
`truncate`	size of truncated tail.
`tail_approx`	Controls whether the geometric tail approximation is used (is faster) or not.
`bound`	control when the geometric tail kicks in; the larger the quicker and less accurate; bound should be larger than 0 and less than 0.001.

Details

The survival function P(L>n) and derived from it also the cdf P(L<=n) and the pmf P(L=n) illustrate the distribution of the EWMA run length...

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

F. F. Gan (1993), An optimal design of EWMA control charts based on the median run length, *J. Stat. Comput. Simulation 45*, 169-184.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

L. A. Jones, C. W. Champ, S. E. Rigdon (2001), The performance of exponentially weighted moving average charts with estimated parameters, *Technometrics 43*, 156-167.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`xewma.sf` for the RL survival function of EWMA control charts w/o pre run uncertainty.

Examples

```
## Jones/Champ/Rigdon (2001)

c4m <- function(m, n) sqrt(2)*gamma( (m*(n-1)+1)/2 )/sqrt( m*(n-1) )/gamma( m*(n-1)/2 )

n <- 5 # sample size

# Figure 6, subfigure r=0.1
lambda <- 0.1
L <- 2.454

CDF0 <- 1 - xewma.sf(lambda, L, 0, 600, sided="two")
m <- 10 # pre run size
CDF1 <- 1 - xewma.sf.prerun(lambda, L/c4m(m,n), 0, 600, sided="two",
size=m, df=m*(n-1), estimated="both")
m <- 20
CDF2 <- 1 - xewma.sf.prerun(lambda, L/c4m(m,n), 0, 600, sided="two",
size=m, df=m*(n-1), estimated="both")
m <- 50
```

```
CDF3 <- 1 - xewma.sf.prerun(lambda, L/c4m(m,n), 0, 600, sided="two",
size=m, df=m*(n-1), estimated="both")

plot(CDF0, type="l", xlab="t", ylab=expression(P(T<=t)), xlim=c(0,500), ylim=c(0,1))
abline(v=0, h=c(0,1), col="grey", lwd=.7)
points((1:5)*100, CDF0[(1:5)*100], pch=18)
lines(CDF1, col="blue")
points((1:5)*100, CDF1[(1:5)*100], pch=2, col="blue")
lines(CDF2, col="red")
points((1:5)*100, CDF2[(1:5)*100], pch=16, col="red")
lines(CDF3, col="green")
points((1:5)*100, CDF3[(1:5)*100], pch=5, col="green")

legend("bottomright", c("Known", "m=10, n=5", "m=20, n=5", "m=50, n=5"),
       col=c("black", "blue", "red", "green"), pch=c(18, 2, 16, 5), lty=1)
```

---

xgrsr.ad *Compute steady-state ARLs of Shiryaev-Roberts schemes*

Description

Computation of the steady-state Average Run Length (ARL) for Shiryaev-Roberts schemes monitoring normal mean.

Usage

```
xgrsr.ad(k, g, mu1, mu0 = 0, zr = 0, sided = "one", MPT = FALSE, r = 30)
```

Arguments

`k`	reference value of the Shiryaev-Roberts scheme.
`g`	control limit (alarm threshold) of Shiryaev-Roberts scheme.
`mu1`	out-of-control mean.
`mu0`	in-control mean.
`zr`	reflection border to enable the numerical algorithms used here.
`sided`	distinguishes between one- and two-sided schemes by choosing `"one"` and `"two"`, respectively. Currently only one-sided schemes are implemented.
`MPT`	switch between the old implementation (`FALSE`) and the new one (`TRUE`) that considers the completed likelihood ratio. MPT contains the initials of G. Moustakides, A. Polunchenko and A. Tartakovsky.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1`.

Details

`xgrsr.ad` determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

Value

Returns a single value which resembles the steady-state ARL.

Author(s)

Sven Knoth

References

S. Knoth (2006), The art of evaluating monitoring schemes – how to measure the performance of control charts? S. Lenz, H. & Wilrich, P. (ed.), *Frontiers in Statistical Quality Control 8*, Physica Verlag, Heidelberg, Germany, 74-99.

G. Moustakides, A. Polunchenko, A. Tartakovsky (2009), Numerical comparison of CUSUM and Shiryaev-Roberts procedures for detectin changes in distributions, *Communications in Statistics: Theory and Methods 38*, 3225-3239.

See Also

`xewma.arl` and `xcusum-arl` for zero-state ARL computation of EWMA and CUSUM control charts, respectively, and `xgrsr.arl` for the zero-state ARL.

Examples

```
## Small study to identify appropriate reflection border to mimic unreflected schemes
k <- .5
g <- log(390)
zrs <- -(0:10)
ZRxgrsr.ad <- Vectorize(xgrsr.ad, "zr")
ads <- ZRxgrsr.ad(k, g, 0, zr=zrs)
data.frame(zrs, ads)

## Table 2 from Knoth (2006)
## original values are
#  mu   arl
#  0    689
#  0.5  30
#  1    8.9
#  1.5  5.1
#  2    3.6
#  2.5  2.8
#  3    2.4
#
k <- .5
g <- log(390)
zr <- -5 # see first example
mus <- (0:6)/2
Mxgrsr.ad <- Vectorize(xgrsr.ad, "mu1")
ads <- round(Mxgrsr.ad(k, g, mus, zr=zr), digits=1)
data.frame(mus, ads)

## Table 4 from Moustakides et al. (2009)
```

```
## original values are
# gamma  A        STADD/steady-state ARL
# 50     28.02    4.37
# 100    56.04    5.46
# 500    280.19   8.33
# 1000   560.37   9.64
# 5000   2801.75  12.79
# 10000  5603.7   14.17
Gxgrsr.ad  <- Vectorize("xgrsr.ad", "g")
As <- c(28.02, 56.04, 280.19, 560.37, 2801.75, 5603.7)
gs <- log(As)
theta <- 1
zr <- -6
ads <- round(Gxgrsr.ad(theta/2, gs, theta, zr=zr, r=100), digits=2)
data.frame(As, ads)
```

---

xgrsr.arl *Compute (zero-state) ARLs of Shiryaev-Roberts schemes*

Description

Computation of the (zero-state) Average Run Length (ARL) for Shiryaev-Roberts schemes monitoring normal mean.

Usage

```
xgrsr.arl(k, g, mu, zr = 0, hs=NULL, sided = "one", q = 1, MPT = FALSE, r = 30)
```

Arguments

k	reference value of the Shiryaev-Roberts scheme.
g	control limit (alarm threshold) of Shiryaev-Roberts scheme.
mu	true mean.
zr	reflection border to enable the numerical algorithms used here.
hs	so-called headstart (enables fast initial response). If `hs=NULL`, then the classical headstart -Inf is used (corresponds to 0 for the non-log scheme).
sided	distinguishes between one- and two-sided schemes by choosing `"one"` and `"two"`, respectively. Currently only one-sided schemes are implemented.
q	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu! = 0$ conditional delays, that is, $E_q(L - q + 1 \| L \geq q)$, will be determined. Note that mu0=0 is implicitely fixed.
MPT	switch between the old implementation (`FALSE`) and the new one (`TRUE`) that considers the complete likelihood ratio. MPT stands for the initials of G. Moustakides, A. Polunchenko and A. Tartakovsky.
r	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1`.

Details

`xgrsr.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

Value

Returns a vector of length `q` which resembles the ARL and the sequence of conditional expected delays for `q=1` and `q>1`, respectively.

Author(s)

Sven Knoth

References

S. Knoth (2006), The art of evaluating monitoring schemes – how to measure the performance of control charts? S. Lenz, H. & Wilrich, P. (ed.), *Frontiers in Statistical Quality Control 8*, Physica Verlag, Heidelberg, Germany, 74-99.

G. Moustakides, A. Polunchenko, A. Tartakovsky (2009), Numerical comparison of CUSUM and Shiryaev-Roberts procedures for detecting changes in distributions, *Communications in Statistics: Theory and Methods 38*, 3225-3239.

See Also

`xewma.arl` and `xcusum-arl` for zero-state ARL computation of EWMA and CUSUM control charts, respectively, and `xgrsr.ad` for the steady-state ARL.

Examples

```
## Small study to identify appropriate reflection border to mimic unreflected schemes
k <- .5
g <- log(390)
zrs <- -(0:10)
ZRxgrsr.arl <- Vectorize(xgrsr.arl, "zr")
arls <- ZRxgrsr.arl(k, g, 0, zr=zrs)
data.frame(zrs, arls)

## Table 2 from Knoth (2006)
## original values are
#  mu   arl
#  0    697
#  0.5  33
#  1    10.4
#  1.5  6.2
#  2    4.4
#  2.5  3.5
#  3    2.9
#
k <- .5
g <- log(390)
```

```
zr <- -5 # see first example
mus <- (0:6)/2
Mxgrsr.arl <- Vectorize(xgrsr.arl, "mu")
arls <- round(Mxgrsr.arl(k, g, mus, zr=zr), digits=1)
data.frame(mus, arls)

XGRSR.arl  <- Vectorize("xgrsr.arl", "g")
zr <- -6

## Table 2 from Moustakides et al. (2009)
## original values are
# gamma   A     ARL/E_infty(L) SADD/E_1(L)
#   50   47.17      50.29        41.40
#  100   94.34     100.28        72.32
#  500  471.70     500.28       209.44
# 1000  943.41    1000.28       298.50
# 5000 4717.04    5000.24       557.87
#10000 9434.08   10000.17       684.17

theta <- .1
As2 <- c(47.17, 94.34, 471.7, 943.41, 4717.04, 9434.08)
gs2 <- log(As2)
arls0 <- round(XGRSR.arl(theta/2, gs2, 0, zr=-5, r=300, MPT=TRUE), digits=2)
arls1 <- round(XGRSR.arl(theta/2, gs2, theta, zr=-5, r=300, MPT=TRUE), digits=2)
data.frame(As2, arls0, arls1)

## Table 3 from Moustakides et al. (2009)
## original values are
# gamma   A     ARL/E_infty(L) SADD/E_1(L)
#   50   37.38      49.45        12.30
#  100   74.76      99.45        16.60
#  500  373.81     499.45        28.05
# 1000  747.62     999.45        33.33
# 5000 3738.08    4999.45        45.96
#10000 7476.15    9999.24        51.49

theta <- .5
As3 <- c(37.38, 74.76, 373.81, 747.62, 3738.08, 7476.15)
gs3 <- log(As3)
arls0 <- round(XGRSR.arl(theta/2, gs3, 0, zr=-5, r=70, MPT=TRUE), digits=2)
arls1 <- round(XGRSR.arl(theta/2, gs3, theta, zr=-5, r=70, MPT=TRUE), digits=2)
data.frame(As3, arls0, arls1)

## Table 4 from Moustakides et al. (2009)
## original values are
# gamma   A     ARL/E_infty(L) SADD/E_1(L)
#   50   28.02      49.78         4.98
#  100   56.04      99.79         6.22
#  500  280.19     499.79         9.30
# 1000  560.37     999.79        10.66
# 5000 2801.85    5000.93        13.86
#10000 5603.70    9999.87        15.24
```

```
theta <- 1
As4 <- c(28.02, 56.04, 280.19, 560.37, 2801.85, 5603.7)
gs4 <- log(As4)
arls0 <- round(XGRSR.arl(theta/2, gs4, 0, zr=-6, r=40, MPT=TRUE), digits=2)
arls1 <- round(XGRSR.arl(theta/2, gs4, theta, zr=-6, r=40, MPT=TRUE), digits=2)
data.frame(As4, arls0, arls1)
```

---

xgrsr.crit *Compute alarm thresholds for Shiryaev-Roberts schemes*

Description

Computation of the alarm thresholds (alarm limits) for Shiryaev-Roberts schemes monitoring normal mean.

Usage

```
xgrsr.crit(k, L0, mu0 = 0, zr = 0, hs = NULL, sided = "one", MPT = FALSE, r = 30)
```

Arguments

k	reference value of the Shiryaev-Roberts scheme.
L0	in-control ARL.
mu0	in-control mean.
zr	reflection border to enable the numerical algorithms used here.
hs	so-called headstart (enables fast initial response). If `hs=NULL`, then the classical headstart -Inf is used (corresponds to 0 for the non-log scheme).
sided	distinguishes between one- and two-sided schemes by choosing `"one"` and `"two"`, respectively. Currently only one-sided schemes are implemented.
MPT	switch between the old implementation (`FALSE`) and the new one (`TRUE`) that considers the completed likelihood ratio. MPT contains the initials of G. Moustakides, A. Polunchenko and A. Tartakovsky.
r	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1`.

Details

`xgrsr.crit` determines the alarm threshold (alarm limit) for given in-control ARL `L0` by applying secant rule and using `xgrsr.arl()`.

Value

Returns a single value which resembles the alarm limit g.

Author(s)

Sven Knoth

References

G. Moustakides, A. Polunchenko, A. Tartakovsky (2009), Numerical comparison of CUSUM and Shiryaev-Roberts procedures for detecting changes in distributions, *Communications in Statistics: Theory and Methods 38*, 3225-3239.r.

See Also

`xgrsr.arl` for zero-state ARL computation.

Examples

```
## Table 4 from Moustakides et al. (2009)
## original values are
# gamma/L0  A/exp(g)
# 50        28.02
# 100       56.04
# 500       280.19
# 1000      560.37
# 5000      2801.75
# 10000     5603.7
theta <- 1
zr <- -6
r <- 100
Lxgrsr.crit  <- Vectorize("xgrsr.crit", "L0")
L0s <- c(50, 100, 500, 1000, 5000, 10000)
gs <- Lxgrsr.crit(theta/2, L0s, zr=zr, r=r)
data.frame(L0s, gs, A=round(exp(gs), digits=2))
```

`xsewma.arl` *Compute ARLs of simultaneous EWMA control charts (mean and variance charts)*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of simultaneous EWMA control charts (based on the sample mean and the sample variance S^2) monitoring normal mean and variance.

Usage

```
xsewma.arl(lx, cx, ls, csu, df, mu, sigma, hsx=0, Nx=40, csl=0,
hss=1, Ns=40, s2.on=TRUE, sided="upper", qm=30)
```

Arguments

`lx`	smoothing parameter lambda of the two-sided mean EWMA chart.
`cx`	control limit of the two-sided mean EWMA control chart.
`ls`	smoothing parameter lambda of the variance EWMA chart.

`csu` upper control limit of the variance EWMA control chart.

`df` actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.

`mu` true mean.

`sigma` true standard deviation.

`hsx` so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.

`Nx` dimension of the approximating matrix of the mean chart.

`csl` lower control limit of the variance EWMA control chart; default value is 0; not considered if `sided` is `"upper"`.

`hss` headstart (enables fast initial response) of the variance chart.

`Ns` dimension of the approximating matrix of the variance chart.

`s2.on` distinguishes between S^2 and S chart.

`sided` distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.

`qm` number of quadrature nodes used for the collocation integrals.

Details

`xsewma.arl` determines the Average Run Length (ARL) by an extension of Gan's (derived from ideas already published by Waldmann) algorithm. The variance EWMA part is treated similarly to the ARL calculation method deployed for the single variance EWMA charts in Knoth (2005), that is, by means of collocation (Chebyshev polynomials). For more details see Knoth (2007).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

K. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *J. R. Stat. Soc., Ser. C, Appl. Stat. 35*, 151-158.

F. F. Gan (1995), Joint monitoring of process mean and variance using exponentially weighted moving average control charts, *Technometrics 37*, 446-453.

S. Knoth (2005), Accurate ARL computation for EWMA-S^2 control charts, *Statistics and Computing 15*, 341-352.

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

See Also

`xewma.arl` and `sewma.arl` for zero-state ARL computation of single mean and variance EWMA control charts, respectively.

Examples

```
## Knoth (2007)
## collocation results in Table 1
## Monte Carlo with 10^9 replicates: 252.307 +/- 0.0078

# process parameters
mu <- 0
sigma <- 1
# subgroup size n=5, df=n-1
df  <- 4
# lambda of mean chart
lx  <- .134
# c_mu^* = .345476571 = cx/sqrt(n) * sqrt(lx/(2-lx)
cx  <- .345476571*sqrt(df+1)/sqrt(lx/(2-lx))
# lambda of variance chart
ls  <- .1
# c_sigma = .477977
csu <- 1 + .477977
# matrix dimensions for mean and variance part
Nx  <- 25
Ns  <- 25
# mode of variance chart
SIDED <- "upper"

arl <- xsewma.arl(lx, cx, ls, csu, df, mu, sigma, Nx=Nx, Ns=Ns, sided=SIDED)
arl
```

---

xsewma.crit — *Compute critical values of simultaneous EWMA control charts (mean and variance charts)*

Description

Computation of the critical values (similar to alarm limits) for different types of simultaneous EWMA control charts (based on the sample mean and the sample variance S^2) monitoring normal mean and variance.

Usage

```
xsewma.crit(lx, ls, L0, df, mu0=0, sigma0=1, cu=NULL, hsx=0,
hss=1, s2.on=TRUE, sided="upper", mode="fixed", Nx=30, Ns=40, qm=30)
```

Arguments

`lx`	smoothing parameter lambda of the two-sided mean EWMA chart.
`ls`	smoothing parameter lambda of the variance EWMA chart.
`L0`	in-control ARL.
`mu0`	in-control mean.
`sigma0`	in-control standard deviation.
`cu`	for two-sided (`sided="two"`) and fixed upper control limit (`mode="fixed"`) a value larger than `sigma0` has to been given, for all other cases `cu` is ignored.
`hsx`	so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.
`hss`	headstart (enables fast initial response) of the variance chart.
`df`	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
`s2.on`	distinguishes between S^2 and S chart.
`sided`	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
`mode`	only deployed for `sided="two"` – with `"fixed"` an upper control limit (see `cu`) is set and only the lower is determined to obtain the in-control ARL `L0`, while with `"unbiased"` a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
`Nx`	dimension of the approximating matrix of the mean chart.
`Ns`	dimension of the approximating matrix of the variance chart.
`qm`	number of quadrature nodes used for the collocation integrals.

Details

`xsewma.crit` determines the critical values (similar to alarm limits) for given in-control ARL `L0` by applying secant rule and using `xsewma.arl()`. In case of `sided="two"` and `mode="unbiased"` a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at `sigma0`. See Knoth (2007) for details and application.

Value

Returns the critical value of the two-sided mean EWMA chart and the lower and upper controls limit `cl` and `cu` of the variance EWMA chart.

Author(s)

Sven Knoth

References

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

See Also

`xsewma.arl` for calculation of ARL of simultaneous EWMA charts.

Examples

```
## Knoth (2007)
## results in Table 2

# subgroup size n=5, df=n-1
df  <- 4
# lambda of mean chart
lx  <- .134
# lambda of variance chart
ls  <- .1
# in-control ARL
L0 <- 252.3
# matrix dimensions for mean and variance part
Nx  <- 25
Ns  <- 25
# mode of variance chart
SIDED <- "upper"

crit <- xsewma.crit(lx, ls, L0, df, sided=SIDED, Nx=Nx, Ns=Ns)
crit

## output as used in Knoth (2007)
crit["cx"]/sqrt(df+1)*sqrt(lx/(2-lx))
crit["cu"] - 1
```

`xsewma.q` *Compute critical values of simultaneous EWMA control charts (mean and variance charts) for given RL quantile*

Description

Computation of the critical values (similar to alarm limits) for different types of simultaneous EWMA control charts (based on the sample mean and the sample variance S^2) monitoring normal mean and variance.

Usage

```
xsewma.q(lx, cx, ls, csu, df, alpha, mu, sigma, hsx=0,
Nx=40, csl=0, hss=1, Ns=40, sided="upper", qm=30)
```

```
xsewma.q.crit(lx, ls, L0, alpha, df, mu0=0, sigma0=1, csu=NULL,
hsx=0, hss=1, sided="upper", mode="fixed", Nx=20, Ns=40, qm=30,
c.error=1e-12, a.error=1e-9)
```

Arguments

lx	smoothing parameter lambda of the two-sided mean EWMA chart.
cx	control limit of the two-sided mean EWMA control chart.
ls	smoothing parameter lambda of the variance EWMA chart.
csu	for two-sided (sided="two") and fixed upper control limit (mode="fixed", only for xsewma.q.crit) a value larger than sigma0 has to been given, for all other cases cu is ignored. It is the upper control limit of the variance EWMA control chart.
L0	in-control RL quantile at level alpha.
df	actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
alpha	quantile level.
mu	true mean.
sigma	true standard deviation.
mu0	in-control mean.
sigma0	in-control standard deviation.
hsx	so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.
Nx	dimension of the approximating matrix of the mean chart.
csl	lower control limit of the variance EWMA control chart; default value is 0; not considered if sided is "upper".
hss	headstart (enables fast initial response) of the variance chart.
Ns	dimension of the approximating matrix of the variance chart.
sided	distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing "upper" (upper chart without reflection at cl – the actual value of of cl is not used).
mode	only deployed for sided="two" – with "fixed" an upper control limit (see cu) is set and only the lower is determined to obtain the in-control ARL L0, while with "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
qm	number of quadrature nodes used for the collocation integrals.
c.error	error bound for two succeeding values of the critical value during applying the secant rule.
a.error	error bound for the quantile level alpha during applying the secant rule.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure and on Knoth (2007). `xsewma.q.crit` determines the critical values (similar to alarm limits) for given in-control RL quantile `L0` at level `alpha` by applying secant rule and using `xsewma.sf()`. In case of `sided="two"` and `mode="unbiased"` a two-dimensional secant rule is applied that also ensures that the maximum of the RL cdf for given standard deviation is attained at `sigma0`.

Value

Returns a single value which resembles the RL quantile of order `alpha` and the critical value of the two-sided mean EWMA chart and the lower and upper controls limit `csl` and `csu` of the variance EWMA chart, respectively.

Author(s)

Sven Knoth

References

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

See Also

`xsewma.arl` for calculation of ARL of simultaneous EWMA charts and `xsewma.sf` for the RL survival function.

Examples

```
## Knoth (2014?)
```

`xsewma.sf` *Compute the survival function of simultaneous EWMA control charts (mean and variance charts)*

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring simultaneously normal mean and variance.

Usage

```
xsewma.sf(n, lx, cx, ls, csu, df, mu, sigma, hsx=0, Nx=40,
csl=0, hss=1, Ns=40, sided="upper", qm=30)
```

Arguments

- `n` calculate sf up to value `n`.
- `lx` smoothing parameter lambda of the two-sided mean EWMA chart.
- `cx` control limit of the two-sided mean EWMA control chart.
- `ls` smoothing parameter lambda of the variance EWMA chart.
- `csu` upper control limit of the variance EWMA control chart.
- `df` actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
- `mu` true mean.
- `sigma` true standard deviation.
- `hsx` so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.
- `Nx` dimension of the approximating matrix of the mean chart.
- `csl` lower control limit of the variance EWMA control chart; default value is 0; not considered if `sided` is `"upper"`.
- `hss` headstart (enables fast initial response) of the variance chart.
- `Ns` dimension of the approximating matrix of the variance chart.
- `sided` distinguishes between one- and two-sided two-sided EWMA-S^2 control charts by choosing `"upper"` (upper chart without reflection at `cl` – the actual value of `cl` is not used), `"Rupper"` (upper chart with reflection at `cl`), `"Rlower"` (lower chart with reflection at `cu`), and `"two"` (two-sided chart), respectively.
- `qm` number of quadrature nodes used for the collocation integrals.

Details

The survival function P(L>n) and derived from it also the cdf P(L<=n) and the pmf P(L=n) illustrate the distribution of the EWMA run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann's survival function iteration procedure and on results in Knoth (2007).

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, *Sequential Analysis 26*, 251-264.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`xsewma.arl` for zero-state ARL computation of simultaneous EWMA control charts.

Examples

```
## Knoth (2014?)
```

`xshewhart.ar1.arl` *Compute ARLs of modified Shewhart control charts for AR(1) data*

Description

Computation of the (zero-state) Average Run Length (ARL) for modified Shewhart charts deployed to the original AR(1) data.

Usage

```
xshewhart.ar1.arl(alpha, cS, delta=0, N1=50, N2=30)
```

Arguments

`alpha`	lag 1 correlation of the data.
`cS`	critical value (alias to alarm limit) of the Shewhart control chart.
`delta`	potential shift in the data (in-control mean is zero.
`N1`	number of quadrature nodes for solving the ARL integral equation, dimension of the resulting linear equation system is `N1`.
`N2`	second number of quadrature nodes for combining the probability density function of the first observation following the margin distribution and the solution of the ARL integral equation.

Details

Following the idea of Schmid (1995), `1- alpha` times the data turns out to be an EWMA smoothing of the underlying AR(1) residuals. Hence, by combining the solution of the EWMA ARL integral equation and the stationary distribution of the AR(1) data (normal distribution is assumed) one gets easily the overall ARL.

Value

It returns a single value resembling the zero-state ARL of a modified Shewhart chart.

Author(s)

Sven Knoth

References

S. Knoth, W. Schmid (2004). Control charts for time series: A review. In *Frontiers in Statistical Quality Control 7*, edited by H.-J. Lenz, P.-T. Wilrich, 210-236, Physica-Verlag.

H. Kramer, W. Schmid (2000). The influence of parameter estimation on the ARL of Shewhart type charts for time series. *Statistical Papers 41*(2), 173-196.

W. Schmid (1995). On the run length of a Shewhart chart for correlated data. *Statistical Papers 36*(1), 111-130.

See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts.

Examples

```
## Table 1 in Kramer/Schmid (2000)

cS <- 3.09023
a  <- seq(0, 4, by=.5)
row1 <- row2 <- row3 <- NULL
for ( i in 1:length(a) ) {
  row1 <- c(row1, round(xshewhart.ar1.arl( 0.4, cS, delta=a[i]), digits=2))
  row2 <- c(row2, round(xshewhart.ar1.arl( 0.2, cS, delta=a[i]), digits=2))
  row3 <- c(row3, round(xshewhart.ar1.arl(-0.2, cS, delta=a[i]), digits=2))
}

results <- rbind(row1, row2, row3)
results

# original values are
# row1 515.44 215.48 61.85 21.63 9.19 4.58 2.61 1.71 1.29
# row2 502.56 204.97 56.72 19.13 7.95 3.97 2.33 1.59 1.25
# row3 502.56 201.41 54.05 17.42 6.89 3.37 2.03 1.46 1.20
```

---

xshewhartrunsrules.arl

Compute ARLs of Shewhart control charts with and without runs rules

---

Description

Computation of the (zero-state and steady-state) Average Run Length (ARL) for Shewhart control charts with and without runs rules monitoring normal mean.

Usage

```
xshewhartrunsrules.arl(mu, c = 1, type = "12")

xshewhartrunsrules.crit(L0, mu = 0, type = "12")
```

```
xshewhartrunsrules.ad(mu1, mu0 = 0, c = 1, type = "12")

xshewhartrunsrules.matrix(mu, c = 1, type = "12")
```

Arguments

`mu`	true mean.
`L0`	pre-defined in-control ARL, that is, determine `c` so that the mean number of observations until a false alarm is `L0`.
`mu1, mu0`	for the steady-state ARL two means are specified, mu0 is the in-control one and usually equal to 0 , and mu1 must be given.
`c`	normalizing constant to ensure specific alarming behavior.
`type`	controls the type of Shewhart chart used, seed details section.

Details

`xshewhartrunsrules.arl` determines the zero-state Average Run Length (ARL) based on the Markov chain approach given in Champ and Woodall (1987). `xshewhartrunsrules.matrix` provides the corresponding transition matrix that is also used in `xDshewhartrunsrules.arl` (ARL for control charting drift). `xshewhartrunsrules.crit` allows to find the normalization constant `c` to attain a fixed in-control ARL. Typically this is needed to calibrate the chart. With `xshewhartrunsrules.ad` the steady-state ARL is calculated. With the argument `type` certain runs rules could be set. The following list gives an overview.

- "1" The classical Shewhart chart with `+/- 3 c sigma` control limits (`c` is typically equal to 1 and can be changed by the argument `c`).
- "12" The classic and the following runs rule: 2 of 3 are beyond `+/- 2 c sigma` on the same side of the chart.
- "13" The classic and the following runs rule: 4 of 5 are beyond `+/- 1 c sigma` on the same side of the chart.
- "14" The classic and the following runs rule: 8 of 8 are on the same side of the chart (referring to the center line).

Value

Returns a single value which resembles the zero-state or steady-state ARL. `xshewhartrunsrules.matrix` returns a matrix.

Author(s)

Sven Knoth

References

C. W. Champ and W. H. Woodall (1987), Exact results for Shewhart control charts with supplementary runs rules, *Technometrics 29*, 393-399.

See Also

`xDshewhartrunsrules.arl` for zero-state ARL of Shewhart control charts with or without runs rules under drift.

Examples

```
## Champ/Woodall (1987)
## Table 1
mus <- (0:15)/5
Mxshewhartrunsrules.arl <- Vectorize(xshewhartrunsrules.arl, "mu")
# standard (1 of 1 beyond 3 sigma) Shewhart chart without runs rules
C1 <- round(Mxshewhartrunsrules.arl(mus, type="1"), digits=2)
# standard + runs rule: 2 of 3 beyond 2 sigma on the same side
C12 <- round(Mxshewhartrunsrules.arl(mus, type="12"), digits=2)
# standard + runs rule: 4 of 5 beyond 1 sigma on the same side
C13 <- round(Mxshewhartrunsrules.arl(mus, type="13"), digits=2)
# standard + runs rule: 8 of 8 on the same side of the center line
C14 <- round(Mxshewhartrunsrules.arl(mus, type="14"), digits=2)

## original results are
#  mus     C1    C12    C13    C14
#  0.0 370.40 225.44 166.05 152.73
#  0.2 308.43 177.56 120.70 110.52
#  0.4 200.08 104.46  63.88  59.76
#  0.6 119.67  57.92  33.99  33.64
#  0.8  71.55  33.12  19.78  21.07
#  1.0  43.89  20.01  12.66  14.58
#  1.2  27.82  12.81   8.84  10.90
#  1.4  18.25   8.69   6.62   8.60
#  1.6  12.38   6.21   5.24   7.03
#  1.8   8.69   4.66   4.33   5.85
#  2.0   6.30   3.65   3.68   4.89
#  2.2   4.72   2.96   3.18   4.08
#  2.4   3.65   2.48   2.78   3.38
#  2.6   2.90   2.13   2.43   2.81
#  2.8   2.38   1.87   2.14   2.35
#  3.0   2.00   1.68   1.89   1.99

data.frame(mus, C1, C12, C13, C14)

## plus calibration, i. e. L0=250 (the maximal value for "14" is 255!
L0 <- 250
c1  <- xshewhartrunsrules.crit(L0, type = "1")
c12 <- xshewhartrunsrules.crit(L0, type = "12")
c13 <- xshewhartrunsrules.crit(L0, type = "13")
c14 <- xshewhartrunsrules.crit(L0, type = "14")
C1  <- round(Mxshewhartrunsrules.arl(mus, c=c1,  type="1"), digits=2)
C12 <- round(Mxshewhartrunsrules.arl(mus, c=c12, type="12"), digits=2)
C13 <- round(Mxshewhartrunsrules.arl(mus, c=c13, type="13"), digits=2)
C14 <- round(Mxshewhartrunsrules.arl(mus, c=c14, type="14"), digits=2)
data.frame(mus, C1, C12, C13, C14)
```

```
## and the steady-state ARL
Mxshewhartrunsrules.ad <- Vectorize(xshewhartrunsrules.ad, "mu1")
C1  <- round(Mxshewhartrunsrules.ad(mus, c=c1,  type="1"), digits=2)
C12 <- round(Mxshewhartrunsrules.ad(mus, c=c12, type="12"), digits=2)
C13 <- round(Mxshewhartrunsrules.ad(mus, c=c13, type="13"), digits=2)
C14 <- round(Mxshewhartrunsrules.ad(mus, c=c14, type="14"), digits=2)
data.frame(mus, C1, C12, C13, C14)
```

`xtcusum.arl` *Compute ARLs of CUSUM control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM control charts monitoring normal mean.

Usage

```
xtcusum.arl(k, h, df, mu, hs = 0, sided="one", mode="tan", r=30)
```

Arguments

`k`	reference value of the CUSUM control chart.
`h`	decision interval (alarm limit, threshold) of the CUSUM control chart.
`df`	degrees of freedom – parameter of the t distribution.
`mu`	true mean.
`hs`	so-called headstart (give fast initial response).
`sided`	distinguish between one- and two-sided CUSUM schemes by choosing `"one"` and `"two"`, respectively.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1`.
`mode`	Controls the type of variables substitution that might improve the numerical performance. Currently, `"identity"`, `"sin"`, `"sinh"`, and `"tan"` (default) are provided.

Details

`xtcusum.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

A. L. Goel, S. M. Wu (1971), Determination of A.R.L. and a contour nomogram for CUSUM charts to control normal mean, *Technometrics 13*, 221-230.

D. Brook, D. A. Evans (1972), An approach to the probability distribution of cusum run length, *Biometrika 59*, 539-548.

J. M. Lucas, R. B. Crosier (1982), Fast initial response for cusum quality-control schemes: Give your cusum a headstart, *Technometrics 24*, 199-205.

L. C. Vance (1986), Average run lengths of cumulative sum control charts for controlling normal means, *Journal of Quality Technology 18*, 189-193.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of one-sided and two-sided CUSUM quality control schemes, *Technometrics 28*, 61-67.

R. B. Crosier (1986), A new two-sided cumulative quality control scheme, *Technometrics 28*, 187-194.

See Also

`xtewma.arl` for zero-state ARL computation of EWMA control charts and `xtcusum.arl` for the zero-state ARL of CUSUM for normal data.

Examples

```
## will follow
```

`xtewma.ad` *Compute steady-state ARLs of EWMA control charts, t distributed data*

Description

Computation of the steady-state Average Run Length (ARL) for different types of EWMA control charts monitoring the mean of t distributed data.

Usage

```
xtewma.ad(l, c, df, mu1, mu0=0, zr=0, z0=0, sided="one", limits="fix",
steady.state.mode="conditional", mode="tan", r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`df`	degrees of freedom – parameter of the t distribution.
`mu1`	in-control mean.
`mu0`	out-of-control mean.
`zr`	reflection border for the one-sided chart.

`z0`	restarting value of the EWMA sequence in case of a false alarm in `steady.state.mode="cyclical"`.
`sided`	distinguishes between one- and two-sided two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`steady.state.mode`	distinguishes between two steady-state modes – conditional and cyclical.
`mode`	Controls the type of variables substitution that might improve the numerical performance. Currently, `"identity"`, `"sin"`, `"sinh"`, and `"tan"` (default) are provided.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

Details

`xtewma.ad` determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature and using the power method for deriving the largest in magnitude eigenvalue and the related left eigenfunction.

Value

Returns a single value which resembles the steady-state ARL.

Author(s)

Sven Knoth

References

R. B. Crosier (1986), A new two-sided cumulative quality control scheme, *Technometrics 28*, 187-194.

S. V. Crowder (1987), A simple method for studying run-length distributions of exponentially weighted moving average charts, *Technometrics 29*, 401-407.

J. M. Lucas and M. S. Saccucci (1990), Exponentially weighted moving average control schemes: Properties and enhancements, *Technometrics 32*, 1-12.

See Also

`xtewma.arl` for zero-state ARL computation and `xewma.ad` for the steady-state ARL for normal data.

Examples

```
## will follow
```

`xtewma.arl` *Compute ARLs of EWMA control charts, t distributed data*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts monitoring the mean of t distributed data.

Usage

```
xtewma.arl(l,c,df,mu,zr=0,hs=0,sided="two",limits="fix",mode="tan",q=1,r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`df`	degrees of freedom – parameter of the t distribution.
`mu`	true mean.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (enables fast initial response).
`sided`	distinguishes between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.
`limits`	distinguishes between different control limits behavior.
`mode`	Controls the type of variables substitution that might improve the numerical performance. Currently, `"identity"`, `"sin"`, `"sinh"`, and `"tan"` (default) are provided.
`q`	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu! = 0$ conditional delays, that is, $E_q(L - q + 1 \vert L \geq q)$, will be determined. Note that mu0=0 is implicitely fixed.
`r`	number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

Details

In case of the EWMA chart with fixed control limits, `xtewma.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature. If `limits` is `"vacl"`, then the method presented in Knoth (2003) is utilized. Other values (normal case) for `limits` are not yet supported.

Value

Except for the fixed limits EWMA charts it returns a single value which resembles the ARL. For fixed limits charts, it returns a vector of length `q` which resembles the ARL and the sequence of conditional expected delays for q=1 and q>1, respectively.

Author(s)

Sven Knoth

References

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

S. V. Crowder (1987), A simple method for studying run-length distributions of exponentially weighted moving average charts, *Technometrics 29*, 401-407.

J. M. Lucas and M. S. Saccucci (1990), Exponentially weighted moving average control schemes: Properties and enhancements, *Technometrics 32*, 1-12.

C. M. Borror, D. C. Montgomery, and G. C. Runger (1999), Robustness of the EWMA control chart to non-normality , *Journal of Quality Technology 31*, 309-316.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts in the normal case.

Examples

```
##  Borror/Montgomery/Runger (1999), Table 3
lambda <- 0.1
cE <- 2.703
df <- c(4, 6, 8, 10, 15, 20, 30, 40, 50)
L0 <- rep(NA, length(df))
for ( i in 1:length(df) ) {
  L0[i] <- round(xtewma.arl(lambda, cE*sqrt(df[i]/(df[i]-2)), df[i], 0), digits=0)
}
data.frame(df, L0)
```

`xtewma.q` *Compute RL quantiles of EWMA control charts*

Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal mean.

Usage

```
xtewma.q(l, c, df, mu, alpha, zr=0, hs=0, sided="two", limits="fix", mode="tan",
q=1, r=40)

xtewma.q.crit(l, L0, df, mu, alpha, zr=0, hs=0, sided="two", limits="fix", mode="tan",
r=40, c.error=1e-12, a.error=1e-9, OUTPUT=FALSE)
```

Arguments

l	smoothing parameter lambda of the EWMA control chart.
c	critical value (similar to alarm limit) of the EWMA control chart.
df	degrees of freedom – parameter of the t distribution.
mu	true mean.
alpha	quantile level.
zr	reflection border for the one-sided chart.
hs	so-called headstart (enables fast initial response).
sided	distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.
limits	distinguishes between different control limits behavior.
mode	Controls the type of variables substitution that might improve the numerical performance. Currently, "identity", "sin", "sinh", and "tan" (default) are provided.
q	change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu! = 0$ conditional delays, that is, $E_q(L - q + 1 \vert L \geq)$, will be determined. Note that mu0=0 is implicitely fixed.
r	number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-sided) or r (two-sided).
L0	in-control quantile value.
c.error	error bound for two succeeding values of the critical value during applying the secant rule.
a.error	error bound for the quantile level alpha during applying the secant rule.
OUTPUT	activate or deactivate additional output.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure. If limits is "vacl", then the method presented in Knoth (2003) is utilized. For details see Knoth (2004).

Value

Returns a single value which resembles the RL quantile of order q.

Author(s)

Sven Knoth

References

F. F. Gan (1993), An optimal design of EWMA control charts based on the median run length, *J. Stat. Comput. Simulation 45*, 169-184.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`xewma.q` for RL quantile computation of EWMA control charts in the normal case.

Examples

```
## will follow
```

`xtewma.sf` *Compute the survival function of EWMA run length*

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal mean.

Usage

```
xtewma.sf(l, c, df, mu, n, zr=0, hs=0, sided="two", limits="fix", mode="tan", q=1, r=40)
```

Arguments

`l`	smoothing parameter lambda of the EWMA control chart.
`c`	critical value (similar to alarm limit) of the EWMA control chart.
`df`	degrees of freedom – parameter of the t distribution.
`mu`	true mean.
`n`	calculate sf up to value `n`.
`zr`	reflection border for the one-sided chart.
`hs`	so-called headstart (enables fast initial response).

`sided` distinguishes between one- and two-sided EWMA control chart by choosing `"one"` and `"two"`, respectively.

`limits` distinguishes between different conrol limits behavior.

`mode` Controls the type of variables substitution that might improve the numerical performance. Currently, `"identity"`, `"sin"`, `"sinh"`, and `"tan"` (default) are provided.

`q` change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state situation for the in-control and out-of-control case, respectively, are calculated. Note that mu0=0 is implicitely fixed.

`r` number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

Details

The survival function P(L>n) and derived from it also the cdf P(L<=n) and the pmf P(L=n) illustrate the distribution of the EWMA run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann's survival function iteration procedure. For varying limits and for change points after 1 the algorithm from Knoth (2004) is applied. For details see Knoth (2004).

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

F. F. Gan (1993), An optimal design of EWMA control charts based on the median run length, *J. Stat. Comput. Simulation 45*, 169-184.

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, *Sequential Analysis 22*, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, *Statistical Papers 46*, 47-64.

K.-H. Waldmann (1986), Bounds for the distribution of the run length of geometric moving average charts, *Appl. Statist. 35*, 151-158.

See Also

`xewma.sf` for survival function computation of EWMA control charts in the normal case.

Examples

```
## will follow
```

`xtshewhart.ar1.arl` *Compute ARLs of modified Shewhart control charts for AR(1) data with Student t residuals*

Description

Computation of the (zero-state) Average Run Length (ARL) for modified Shewhart charts deployed to the original AR(1) data where the residuals follow a Student t distribution.

Usage

```
xtshewhart.ar1.arl(alpha, cS, df, delta=0, N1=50, N2=30, N3=2*N2, INFI=10, mode="tan")
```

Arguments

`alpha`	lag 1 correlation of the data.
`cS`	critical value (alias to alarm limit) of the Shewhart control chart.
`df`	degrees of freedom – parameter of the t distribution.
`delta`	potential shift in the data (in-control mean is zero.
`N1`	number of quadrature nodes for solving the ARL integral equation, dimension of the resulting linear equation system is `N1`.
`N2`	second number of quadrature nodes for combining the probability density function of the first observation following the margin distribution and the solution of the ARL integral equation.
`N3`	third number of quadrature nodes for solving the left eigenfunction integral equation to determine the margin density (see Andel/Hrach, 2000), dimension of the resulting linear equation system is `N3`.
`INFI`	substitute of `Inf` – the left eigenfunction integral equation is defined on the whole real axis; now it is reduced to `(-INFI,INFI)`.
`mode`	Controls the type of variables substitution that might improve the numerical performance. Currently, `"identity"`, `"sin"`, `"sinh"`, and `"tan"` (default) are provided.

Details

Following the idea of Schmid (1995), `1-alpha` times the data turns out to be an EWMA smoothing of the underlying AR(1) residuals. Hence, by combining the solution of the EWMA ARL integral equation and the stationary distribution of the AR(1) data (here Student t distribution is assumed) one gets easily the overall ARL.

Value

It returns a single value resembling the zero-state ARL of a modified Shewhart chart.

Author(s)

Sven Knoth

References

J. Andel, K. Hrach (2000). On calculation of stationary density of autoregressive processes. *Kybernetika, Institute of Information Theory and Automation AS CR 36*(3), 311-319.

H. Kramer, W. Schmid (2000). The influence of parameter estimation on the ARL of Shewhart type charts for time series. *Statistical Papers 41*(2), 173-196.

W. Schmid (1995). On the run length of a Shewhart chart for correlated data. *Statistical Papers 36*(1), 111-130.

See Also

`xtewma.arl` for zero-state ARL computation of EWMA control charts in case of Student `t` distributed data.

Examples

```
## will follow
```

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